

A composite image of the Crab nebula, showing a complex, multi-colored structure with a central pulsar. The image is a mix of blue and red, representing X-ray and optical emissions respectively. The text is overlaid on this image.

*Towards a self-consistent theory
of pulsar magnetospheric
HE/VHE emissions*

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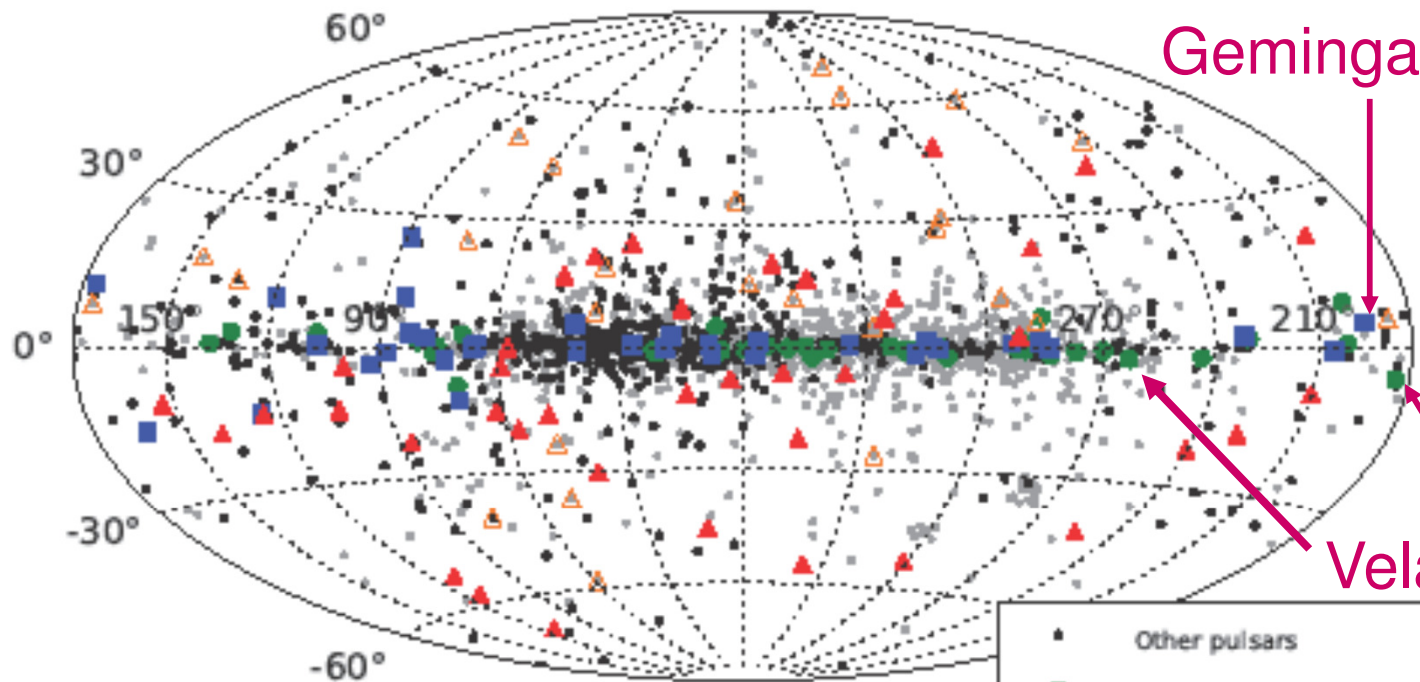
July 26, 2013

Crab nebula: Composite image of X-ray [blue] and optical [red]

§1 γ -ray Pulsar Observations

After 2008, LAT aboard Fermi has detected more than **117** pulsars above 100 MeV.

Fermi/LAT point sources (>100 MeV)



2nd LAT catalog (Abdo+ 2013)

- Other pulsars
- LAT radio-loud pulsar
- LAT radio-quiet pulsar
- △ Radio MSP from LAT UnID
- ▲ LAT millisecond pulsar

Large Area Telescope



Fermi γ -ray space telescope

Geminga

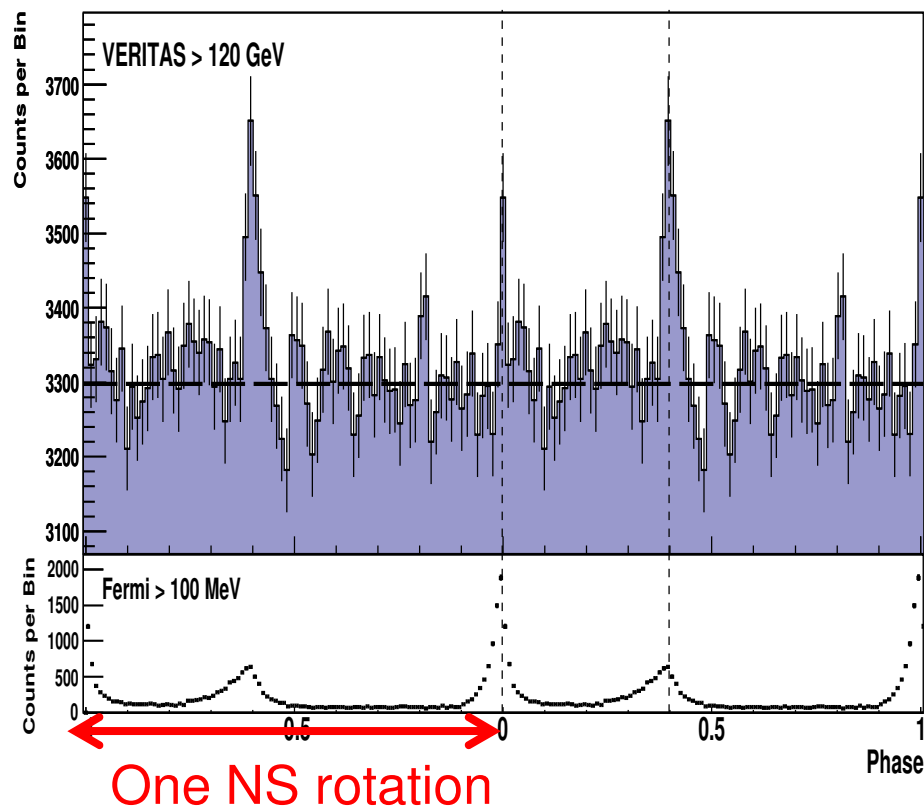
Crab

Vela

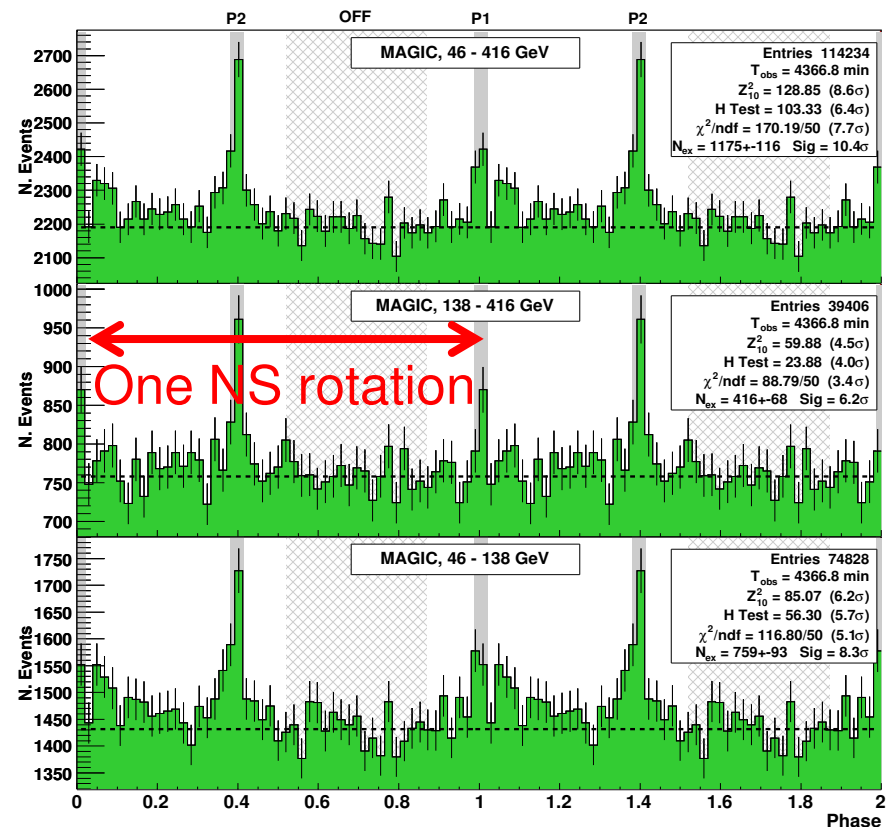
§1 γ -ray Pulsar Observations

Ground-based, Imaging Air Cherenkov Telescopes (IACTs) found pulsed emission above 25 GeV from the Crab pulsar.

VERITAS (> 120 GeV)
Aliu+ (2011, Science 334, 69)



MAGIC (25–416 GeV)
Aleksić+ (2011a,b)

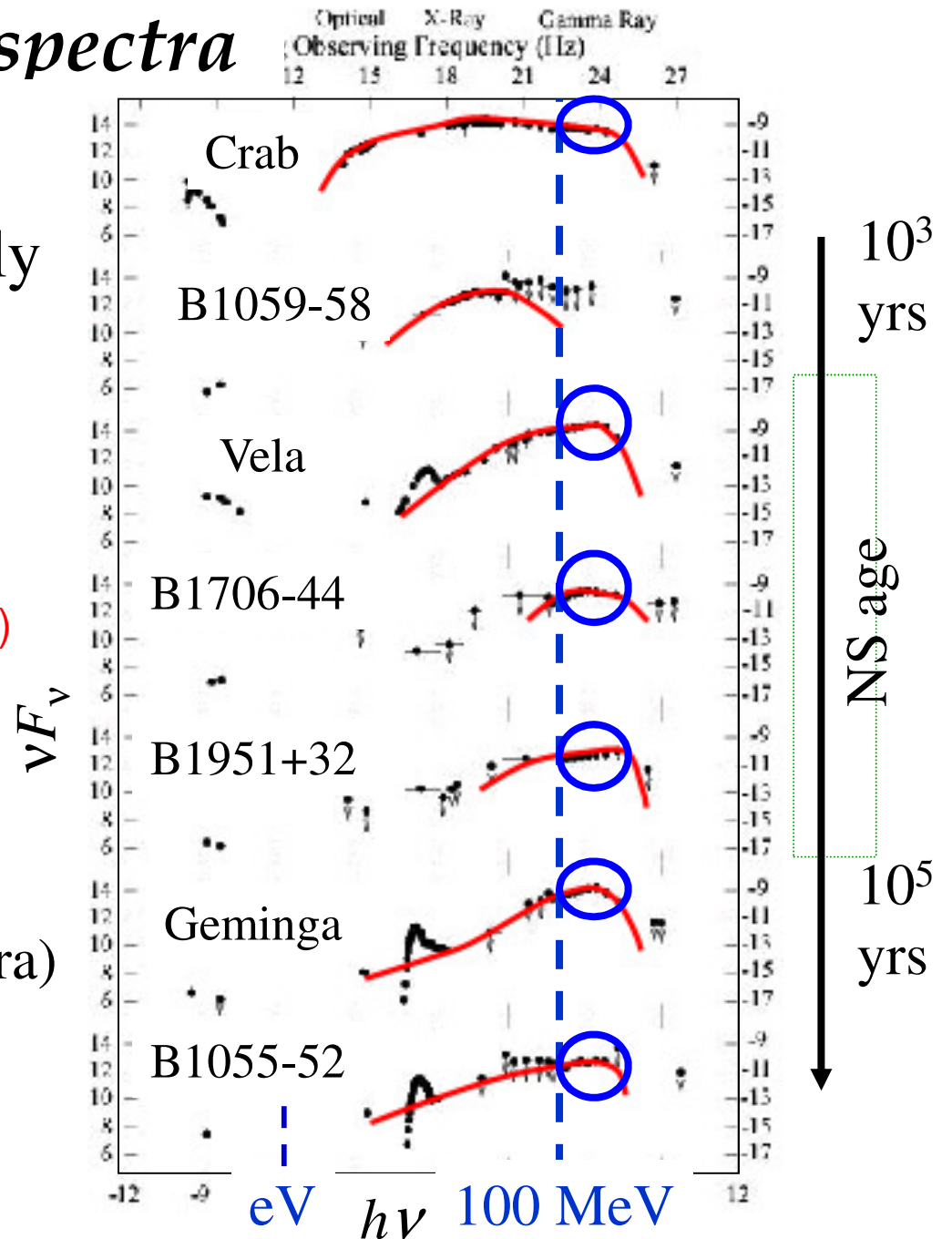


Pulsed broad-band spectra

● High-energy ($\sim \text{GeV}$) photons are emitted mainly via **curvature process** by ultra-relativistic, primary e^- 's/ e^+ 's.

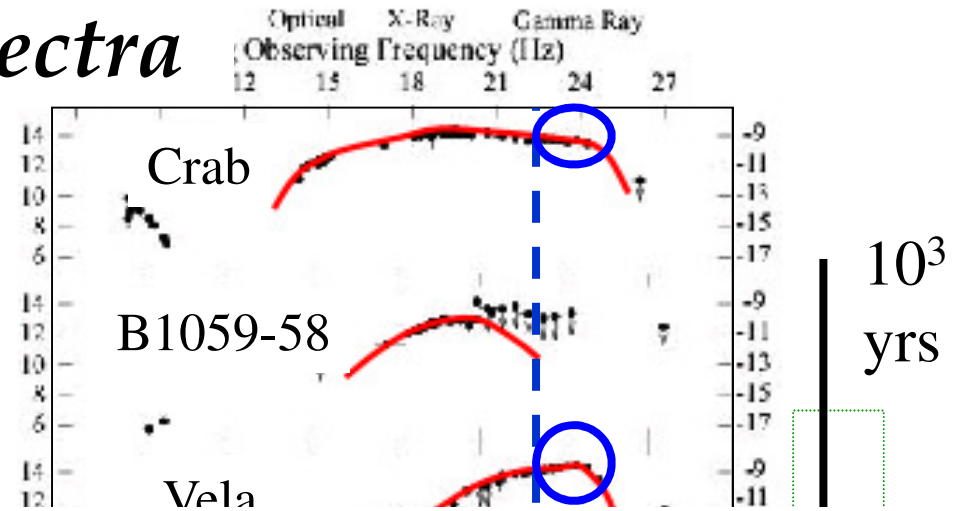
(created in the particle accelerator)

(Thompson, EGRET spectra)



Pulsed broad-band spectra

- High-energy ($> 100\text{MeV}$) photons are emitted mainly via **curvature process** by ultra-relativistic e^\pm 's.



What is the curvature process?

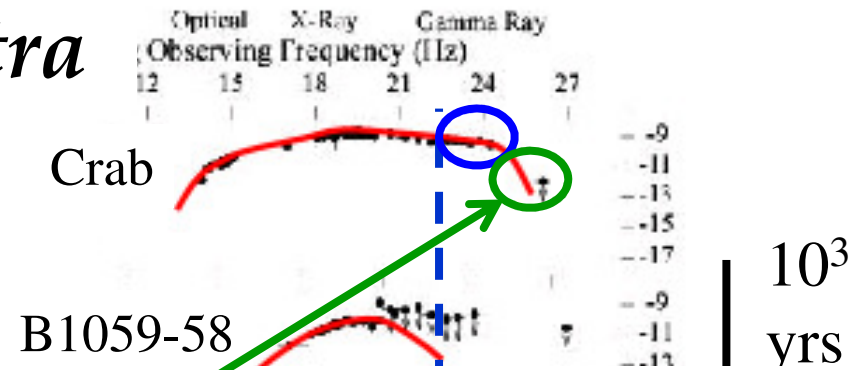
Consider relativistic charges moving along curved \mathbf{B} . They emit **curvature radiation**, provided $P_{\parallel} \gg P_{\perp}$. A charge e with Lorentz factor γ emits the following synchrotron radiation,

$$\begin{aligned} \text{characteristic energy: } \hbar\omega_{\text{curv}} &= \frac{3}{2} \hbar\gamma^3 \frac{c}{R_c} & \text{cf. synchrotron case } \hbar\omega_c &= \frac{3}{2} \hbar\gamma^3 \frac{c}{r_g} \\ \text{radiation power: } P_{\text{curv}} &= \frac{3e^2}{2c^3} \gamma^4 \left(\frac{c^2}{R_c} \right)^2 & P_{\text{synch}} &= \frac{3e^2}{2c^3} \gamma^4 \left(\frac{c^2}{r_g} \right)^2 \end{aligned}$$

Pulsed broad-band spectra

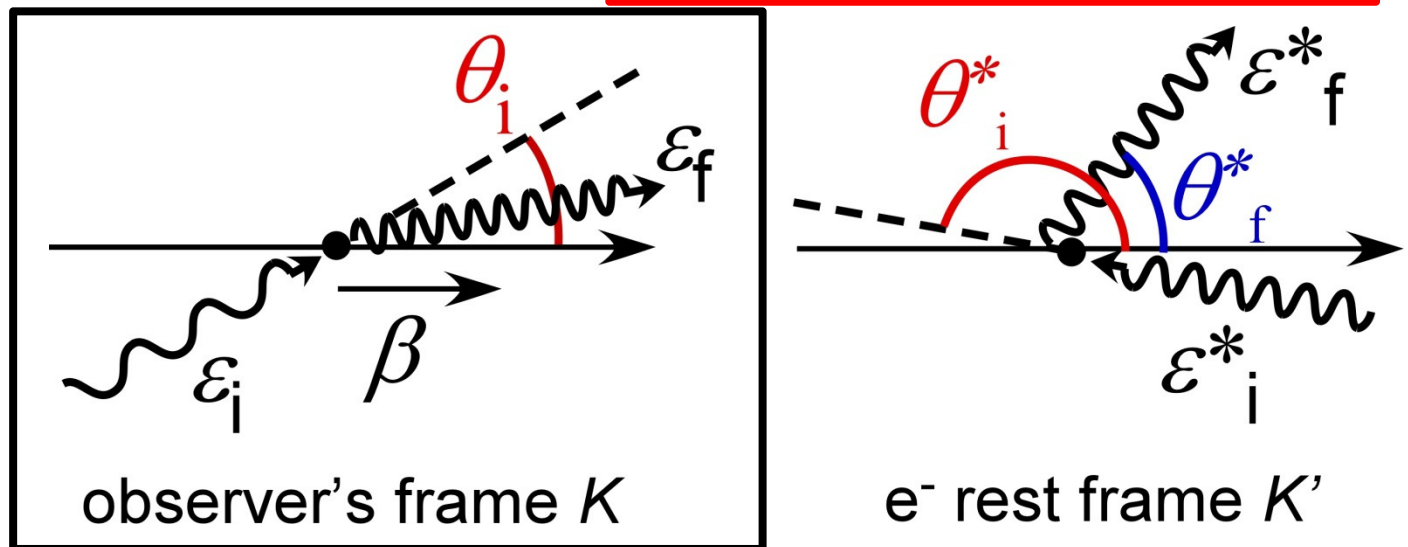
● High-energy ($> 100\text{MeV}$) photons are emitted mainly via **curvature** process by ultra-relativistic, primary e^\pm 's.

● However, $> 20\text{ GeV}$, **ICS by secondary & tertiary e^\pm 's** contributes.



For pulsar VHE emissions, **Klein-Nishina effect** becomes important, because $\mathcal{E}_i^* \gg m_e c^2$.

Fig. Two Lorentz frames when a photon is up-scattered by a relativistic e^- .

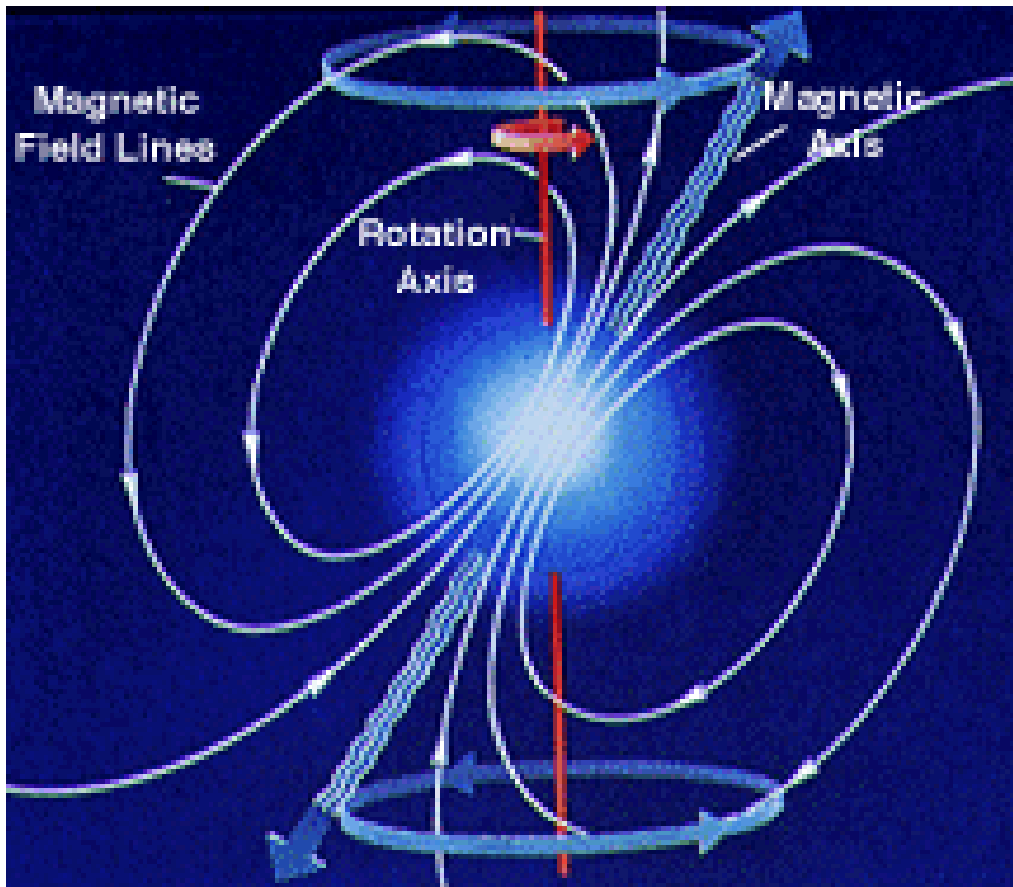


§2 Pulsar Emission Models

Let us begin with considering how and where such incoherent, high-energy photons are emitted from pulsars.

§2 Pulsar Emission Models

High-energy emissions are realized when the **rotational energy of the NS** is electro-dynamically extracted and partly dissipated in the magnetosphere. (e.g., unipolar inductor)



Magnetic and rotation axes are generally misaligned.

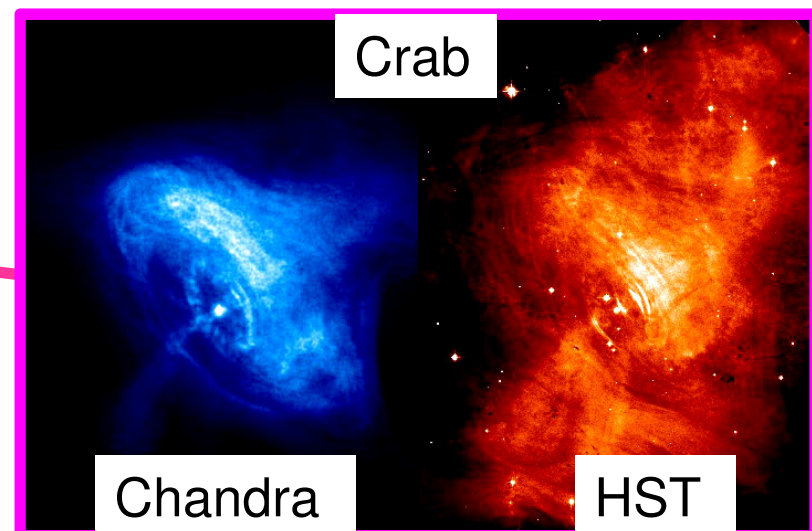
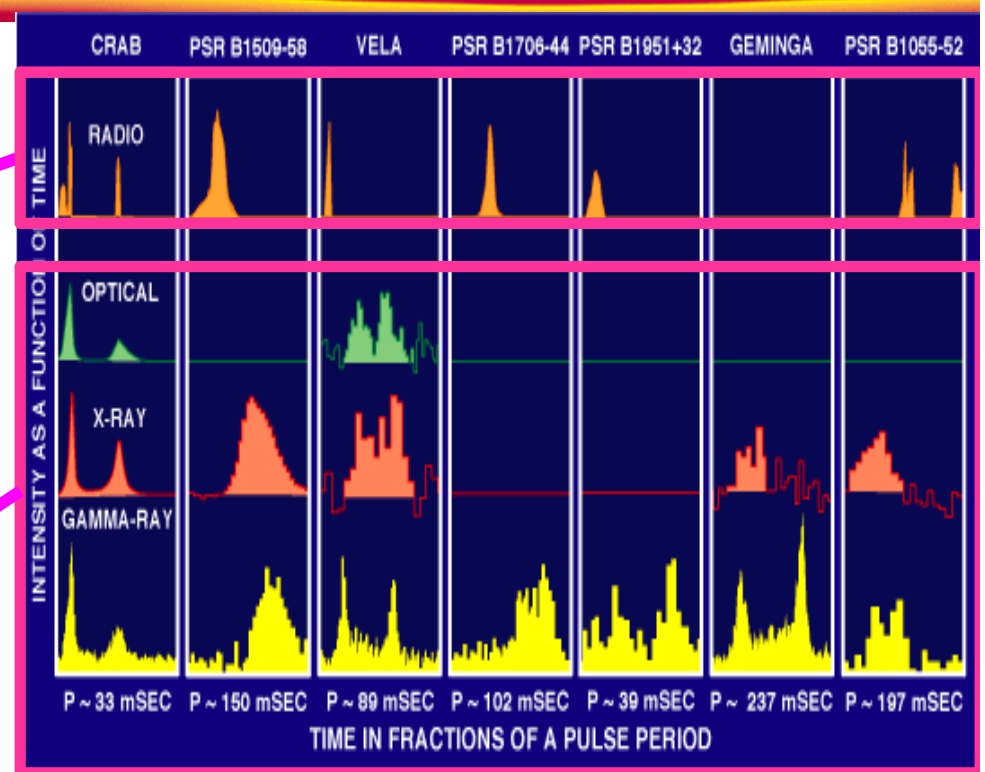
Pulsars:

rapidly rotating, highly magnetized NS

§2 Pulsar Emission Models

Pulsar emission takes place at ...

- **Polar gap**
($r < 30$ km), near NS surface
- **Outer gap, or slot gap**
($r \sim 10^3$ km), near the light cylinder
(outside the null surface)
- **Wind region**
We neglect the emission from the wind region, because they are not pulsed.



§2 Pulsar Emission Models

Pulsar emission takes place at ...

- **Polar gap**
($r < 30$ km), near NS surface
- **Outer gap, or slot gap**
($r \sim 10^3$ km), near the light cylinder
(outside the null surface)

E_{\parallel} arises in a limited volume near the PC surface due to heavy screening by pair discharge.

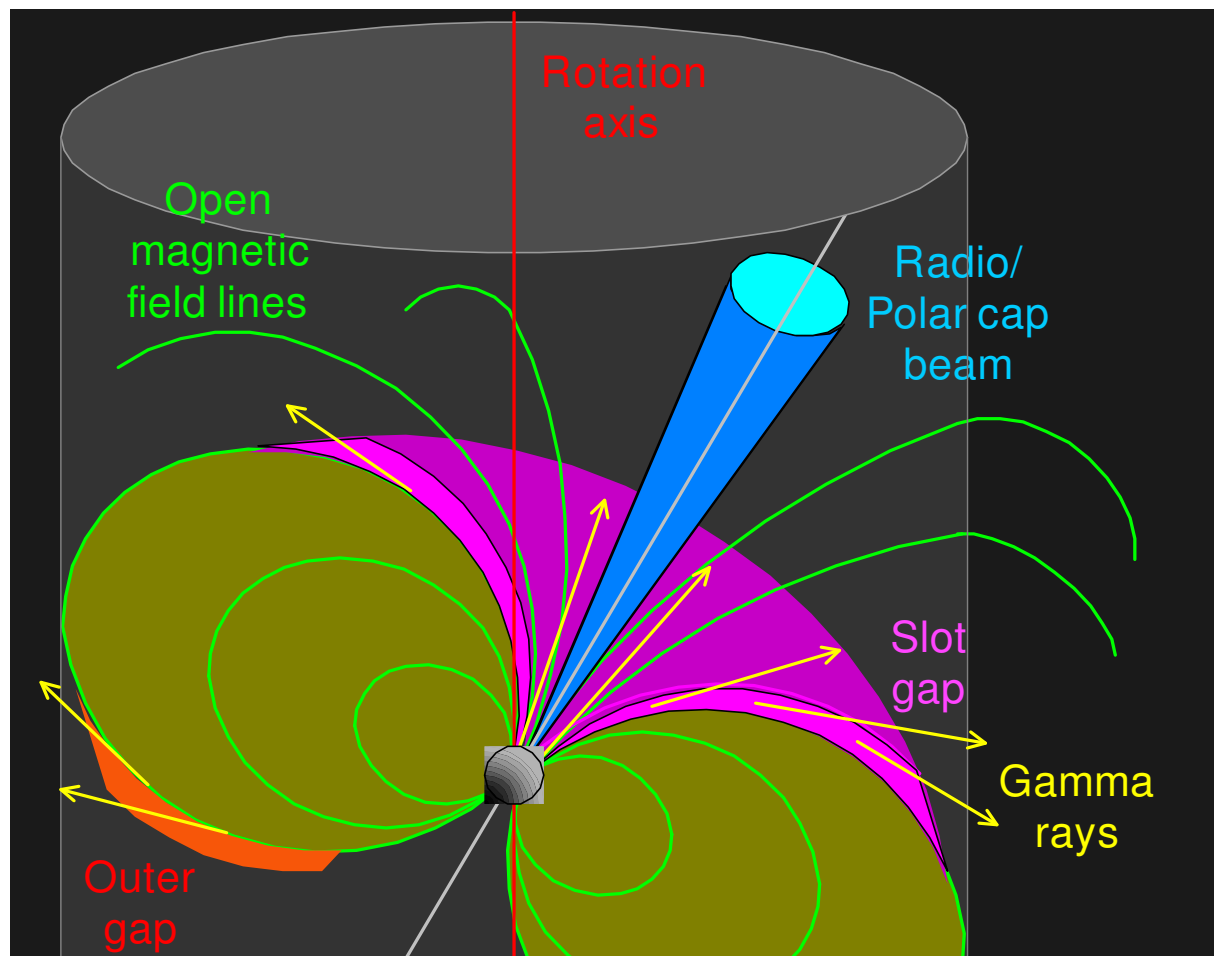
E_{\parallel} arises in a greater volume in the higher altitudes due to less efficient pair production.

§2 Pulsar Emission Models

Early 80's, the **polar-cap (PC) model** was proposed.

(Daugherty & Harding ApJ 252, 337, 1982)

A single PC beam can produce a variety of pulse profiles.



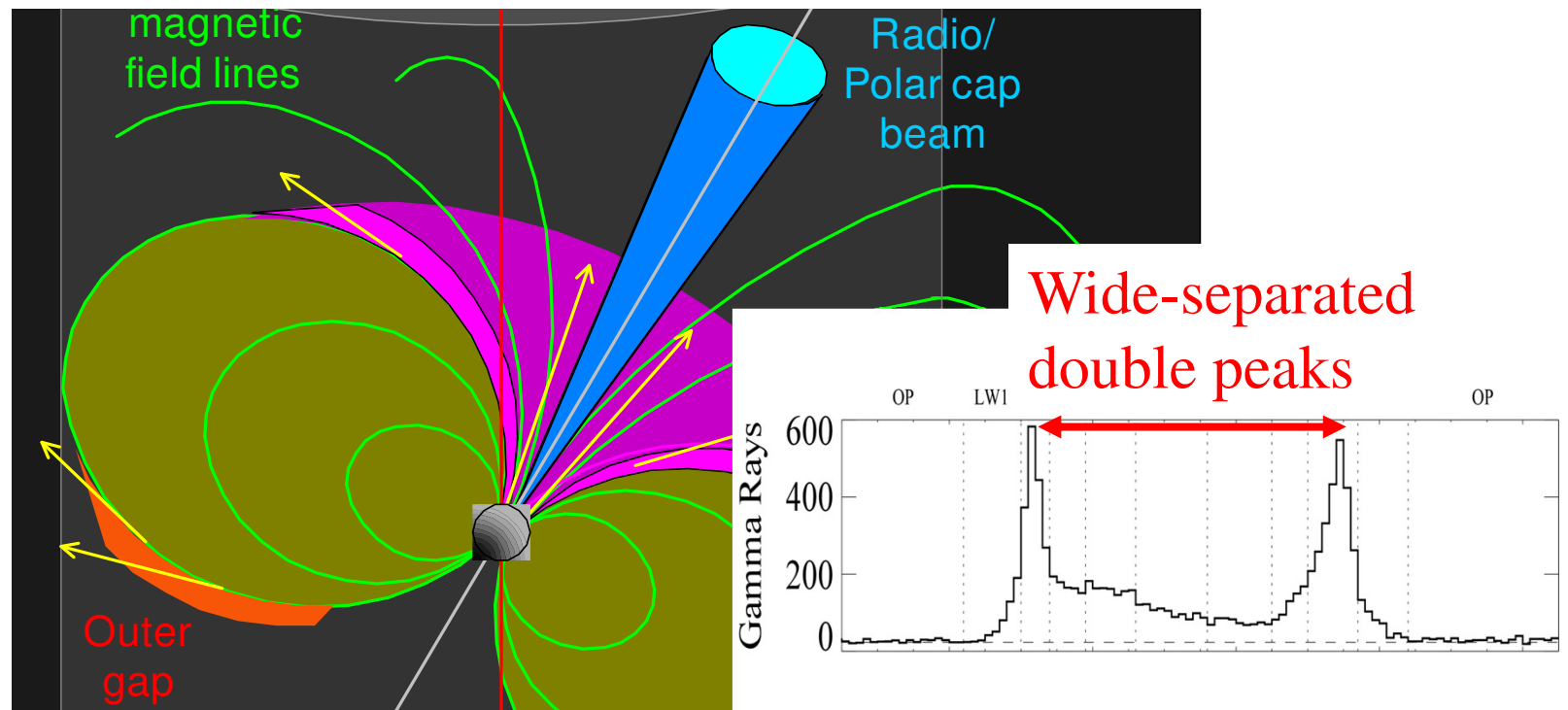
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§2 *Pulsar Emission Models*

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However, the emission solid angle ($\Delta\Omega \ll 1$ ster) was too small to reproduce the wide-separated double peaks.

Thus, a **high-altitude emission** drew attention.

§2 Pulsar Emission Models

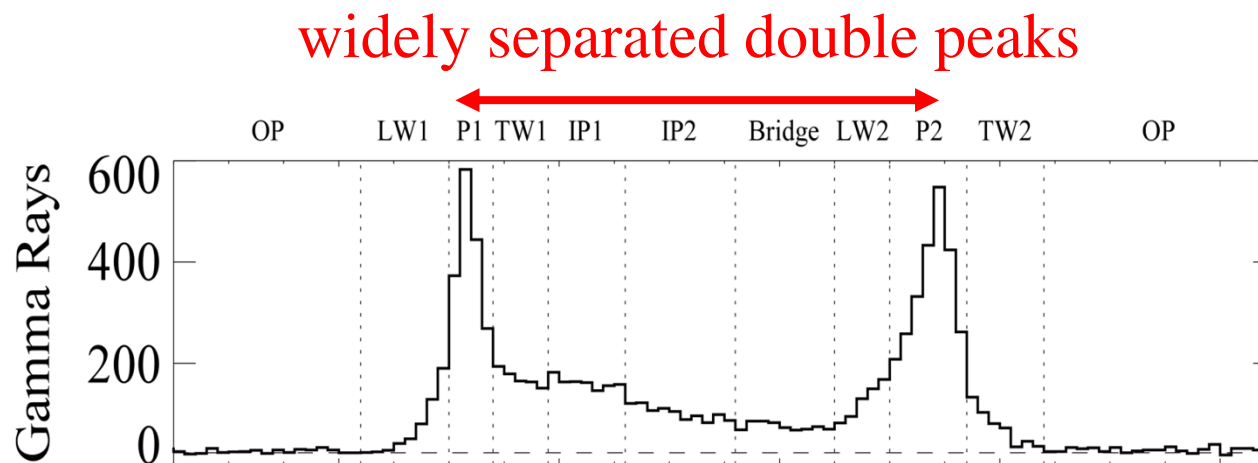
To contrive a higher-altitude emission model, the polar gap was extended into higher altitudes (in fact, by hand).

Muslimov & Harding (2004a, ApJ 606, 1143;
2004b, ApJ 617, 471)

Dyks, Harding & Rudak (2004, ApJ 606, 1125)

Harding+ (2008, ApJ ApJ 680, 1378)

They explained, e.g., the widely separated double peaks.



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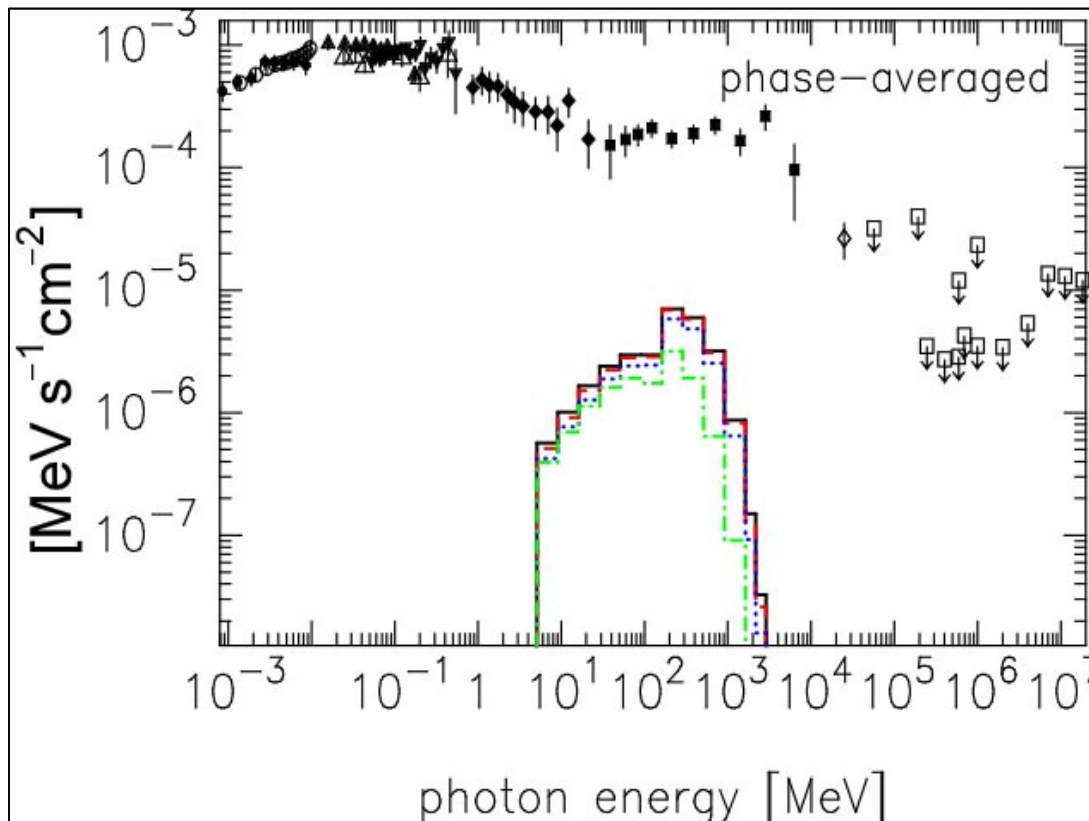
They explained, e.g., the widely separated double peaks.

However, unfortunately, the higher-altitude **SG model** contains two fatal electro-dynamical inconsistencies.

§2 Pulsar Emission Models

Problem 1: insufficient luminosity

This numerical conclusion is confirmed also analytically (KH'08), showing that a **SG can produce only a negligible γ -ray flux.**



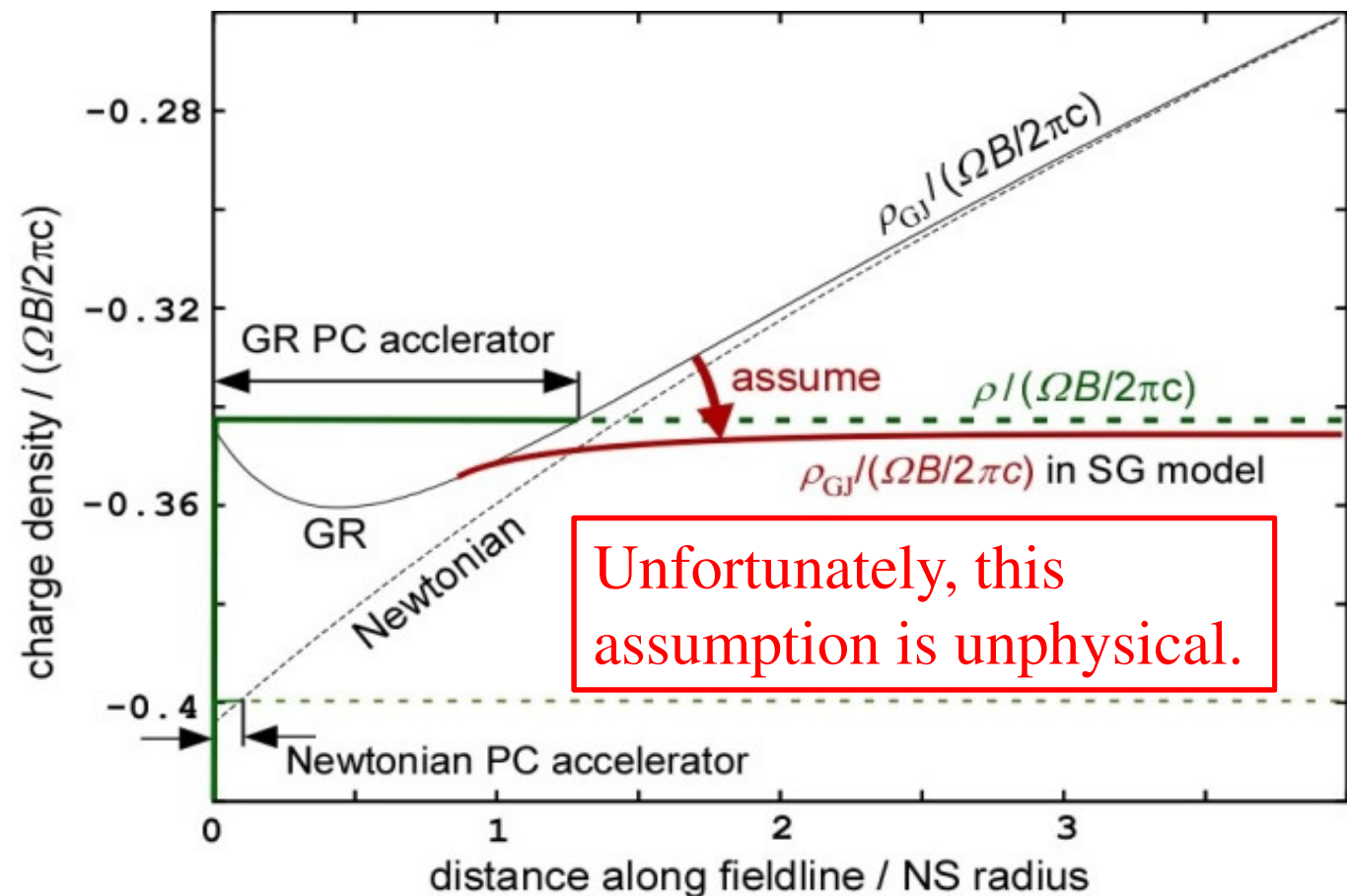
For details, please also refer to my talk last year at CTA-Japan meeting.

KH (2008) ApJ 688, L25

§2 Pulsar Emission Models

Problem #2: unphysical assumption of ρ_{GJ}/B

In the SG model, they assume ρ_{GJ}/B distribution that contradicts with the Maxwell equation.



For details, please also refer to my talk last year at CTA-Japan meeting.

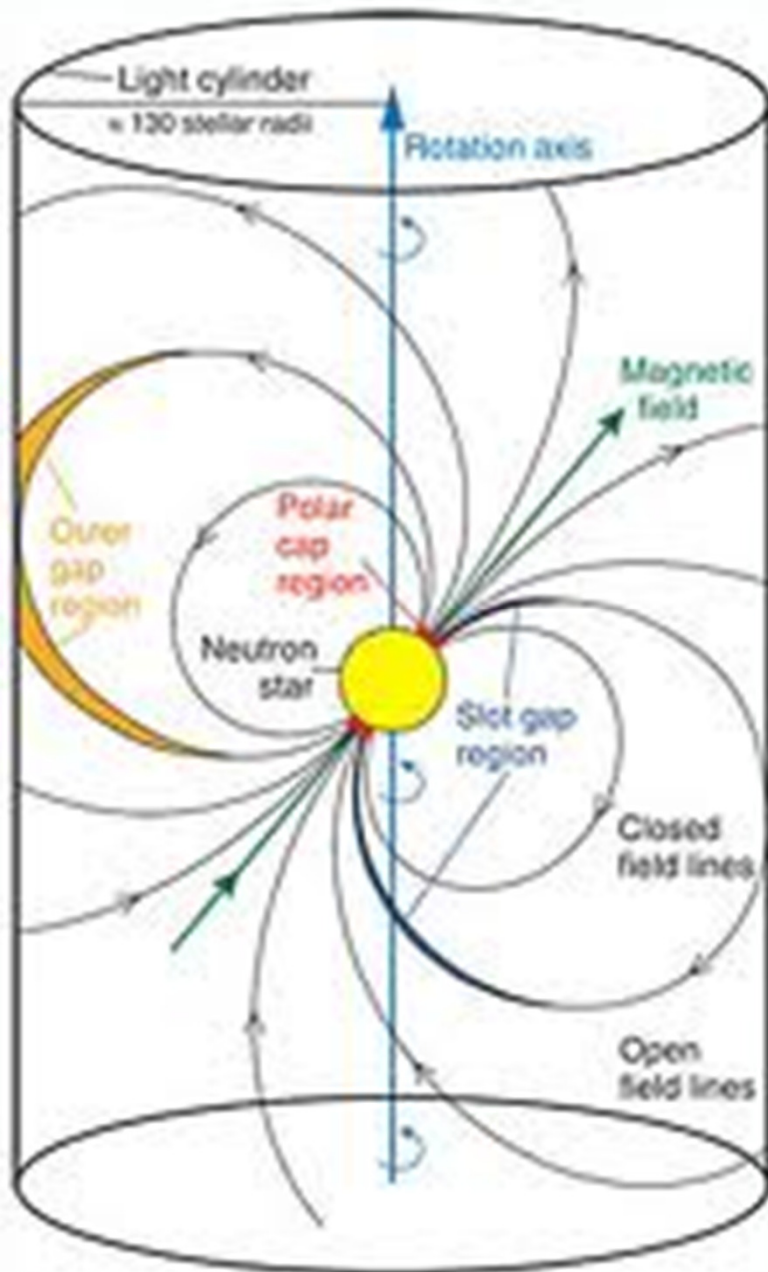
§2 Pulsar Emission Models

Problem #2: unphysical assumption of ρ_{GJ}/B

In fact, to solve the insufficient flux problem (prob. #1), a geometrically thick version of the higher-altitude SG model, the pair-starved PC (PSPC) model, was proposed ([Venter+ 2009, ApJ 707, 800](#)). However, the PSPC model adopts the same ρ_{GJ}/B distribution, which means that the same difficulty applies.

Higher-latitude SG model & PSPC model contradict with Maxwell eq. That is, higher-altitude extension of the polar-cap model failed.

§2 Pulsar Emission Models



As an alternative possibility of high-altitude emission model, the **outer gap model** was proposed.

Cheng, Ho, Ruderman
(1986, ApJ 300, 500)

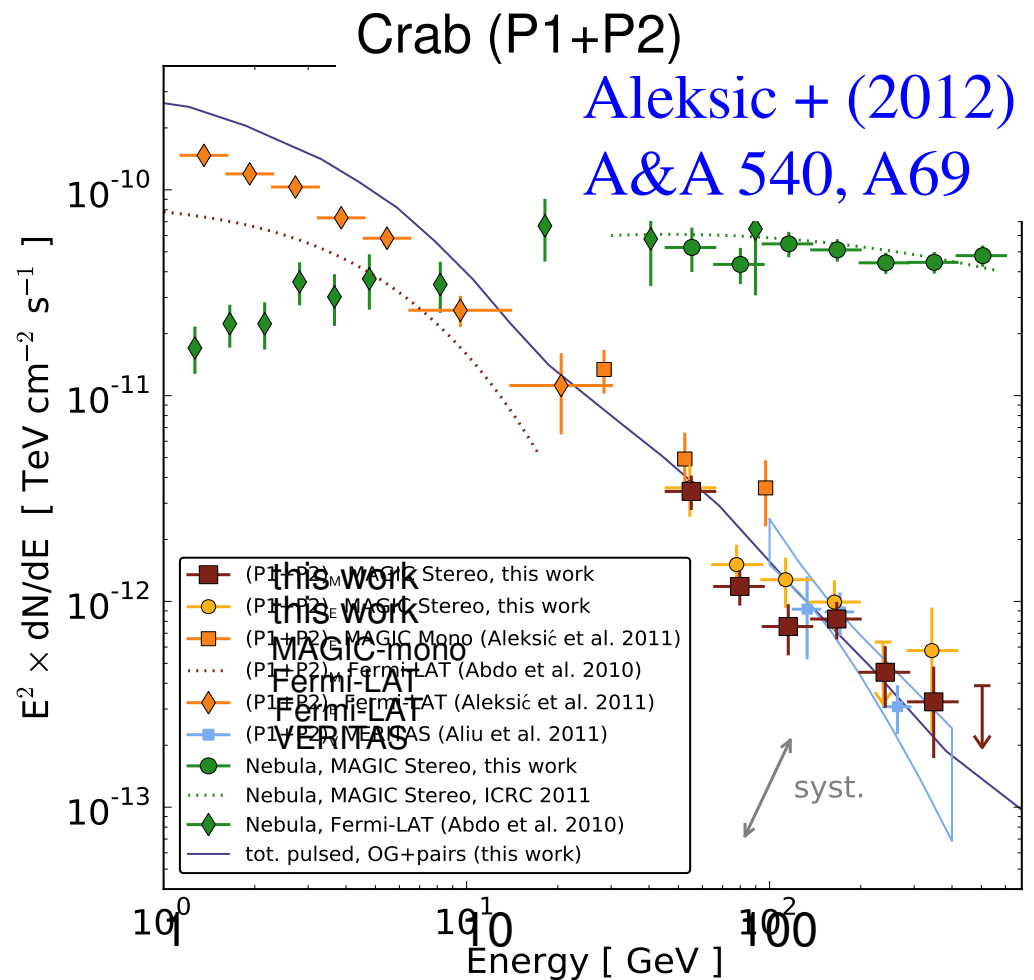
So far, there have been found no serious electro-dynamical problems in the OG model (unlike SG or PSPC model).

Thus, let us concentrate on the OG model in what follows.

§2 Pulsar Emission Models

Indeed, the sub-TeV components from the Crab pulsar shows that pulsed γ -rays are emitted from the **outer magnetosphere** (~~$\gamma B \rightarrow ee$~~).

We thus consider the outer-gap model (Cheng+ 86, ApJ 300,500) in this talk.



§2 *Pulsar Emission Models*

Various attempts have been made on recent OG model:

3-D geometrical model

→ phase-resolved spectra (Cheng + '00; Tang + '08)

→ atlas of light curves for PC, OG, SG models

(Watters + '08)

2-D self-consistent solution

(Takata + '06; KH '06)

3-D self-consistent solution

→ phase-resolved spectra, absolute luminosity

if we give only P , dP/dt , α , kT (+ ζ) (this talk)

In this talk, I'll present the most recent results obtained in my 3-D version of self-consistent OG calculations.

§2 Pulsar Emission Models

3-D self-consistent OG model

Death line of normal and millisecond PSRs on (P, \dot{P}) plane
(Wang & KH 2011, ApJ 736, 127)

Spectral hardening of trailing light-curve peak
(KH 2011, ApJ 733, L49)

Evolution of γ -ray luminosity of rotation-powered PSRs
(KH 2013, ApJ 766, 98)

Today's talk I.

Crab pulsars HE-VHE pulsed emission

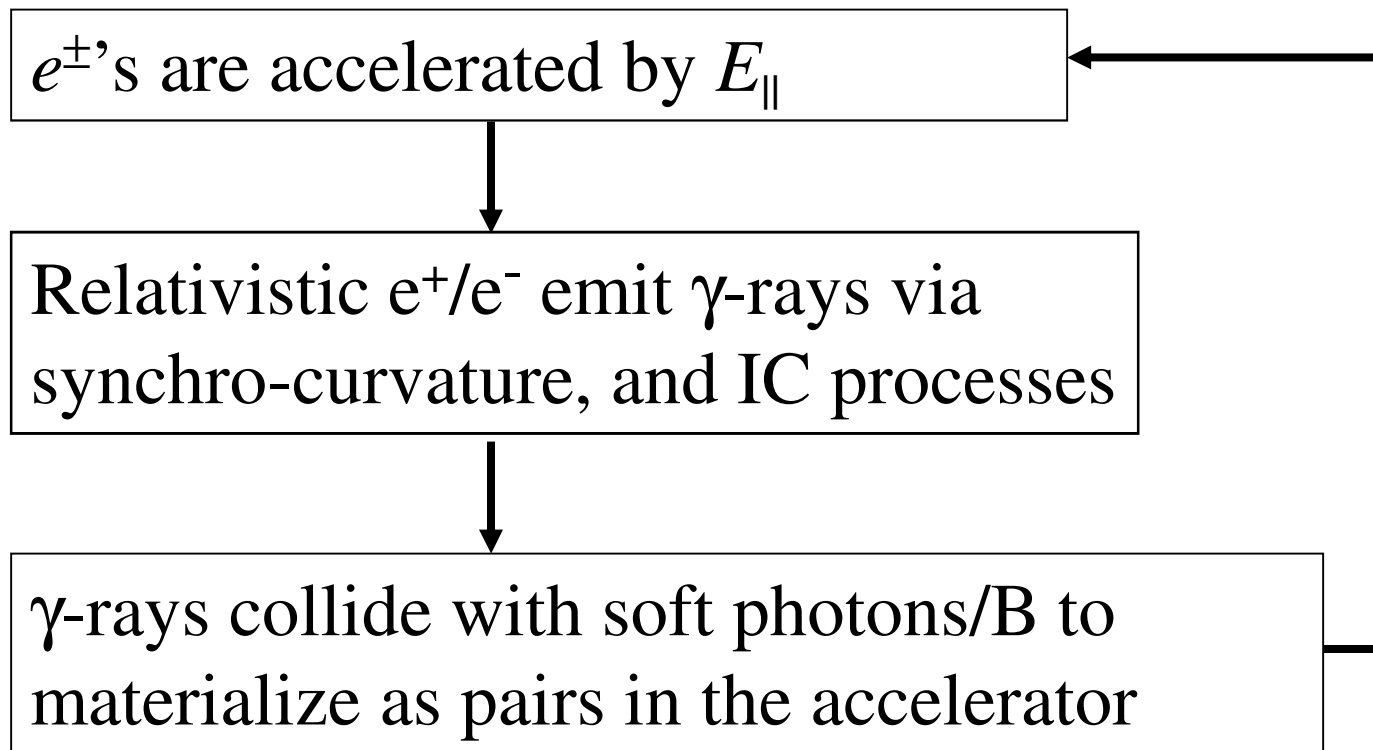
(Alkesic + 2011, ApJ 742, 43; Alkesic + 2012, AA 540, A69)

Today's talk II.

Comment on Lorentz invariance violation tests.

§3 *Modern Outer-gap Model: Formalism*

Self-sustained pair-production cascade in a rotating NS magnetosphere:



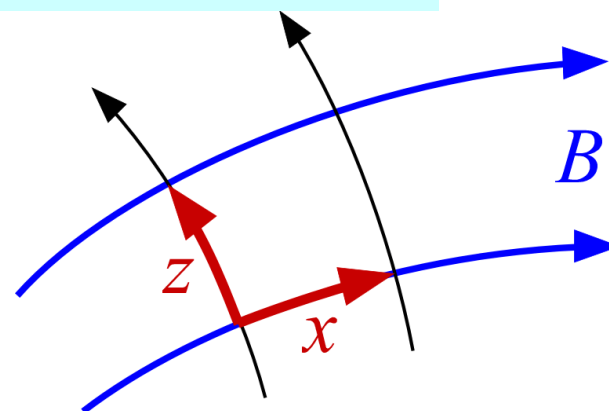
§3 Modern OG Model: Formalism

Poisson equation for electrostatic potential ψ :

$$-\nabla^2\psi = -\frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial y^2} - \frac{\partial^2\psi}{\partial z^2} = 4\pi(\rho - \rho_{\text{GJ}}),$$

where

$$E_{\parallel} \equiv -\frac{\partial\psi}{\partial x}, \quad \rho_{\text{GJ}} \equiv -\frac{\mathbf{\Omega} \cdot \mathbf{B}}{2\pi c},$$



$$\rho(\mathbf{x}) \equiv e \int_1^{\infty} d\gamma \int_0^{\pi} d\chi [N_+(\mathbf{x}, \gamma, \chi) - N_-(\mathbf{x}, \gamma, \chi)] + \rho_{\text{ion}}(\mathbf{x}),$$

$$\mathbf{x} = (x, y, z).$$

N_+/N_- : distrib. func. of e^+/e^-
 γ : Lorentz factor of e^+/e^-
 χ : pitch angle of e^+/e^-

§3 Modern OG Model: Formalism

Assuming $\partial_t + \Omega \partial_\phi = 0$, we solve the e^\pm 's Boltzmann eqs.

$$\frac{\partial N_\pm}{\partial t} + \vec{v} \cdot \nabla N_\pm + \left(e \vec{E}_\parallel + \frac{\vec{v}}{c} \times \vec{B} \right) \cdot \frac{\partial N_\pm}{\partial \vec{p}} = S_{IC} + S_{SC} + \int \alpha_\nu d\nu \int \frac{I_\nu}{h\nu} d\omega$$

together with the radiative transfer equation,

$$\frac{dI_\nu}{dl} = -\alpha_\nu I_\nu + j_\nu$$

N_\pm : positronic/electronic spatial # density,

E_\parallel : magnetic-field-aligned electric field,

S_{IC} : ICS re-distribution function, $d\omega$: solid angle element,

I_ν : specific intensity, l : path length along the ray

α_ν : absorption coefficient, j_ν : emission coefficient

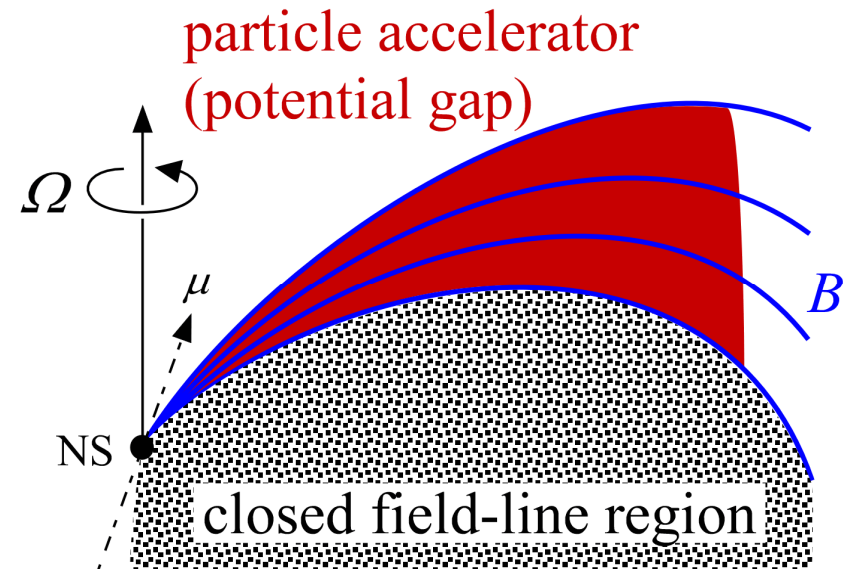
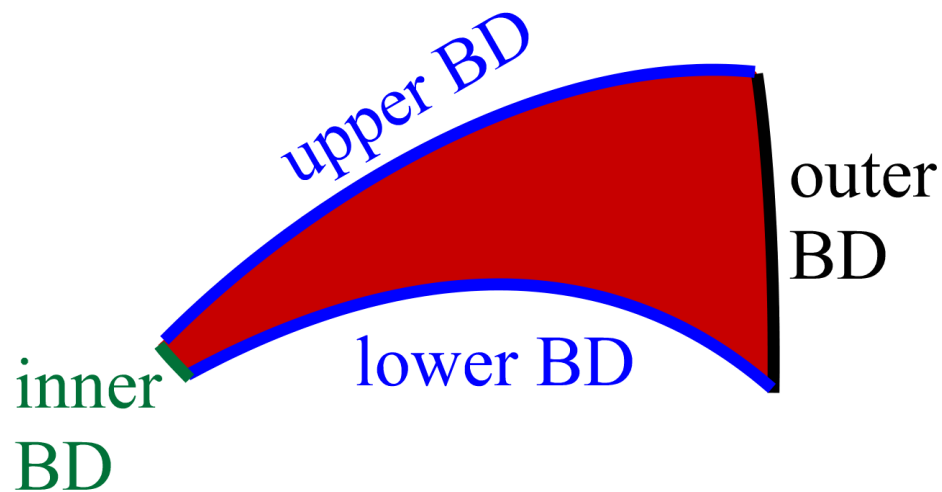
§3 Modern OG Model: Formalism

Boundary Conditions

To solve the elliptic-type differential eq. (Poisson eq.), we impose

$$\Psi = 0 \quad \text{at inner, lower, upper BDs}$$

$$\frac{\partial \Psi}{\partial x} = 0 \quad \text{at outer BD}$$



§3 Modern OG Model: Formalism

At the inner BD

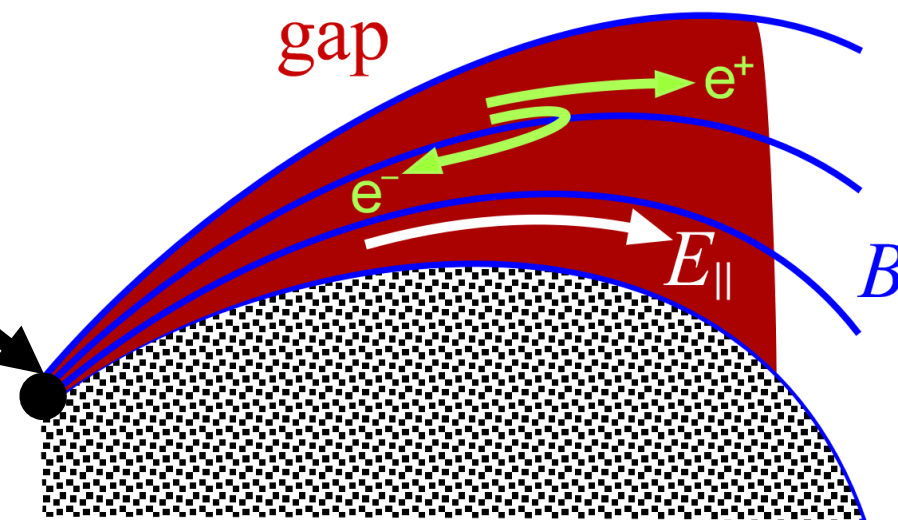
To solve the hyperbolic-type PDE (e^\pm Boltzmann eq.), we impose

$$N_+(x^{\text{in}}, z, \gamma) = 0$$

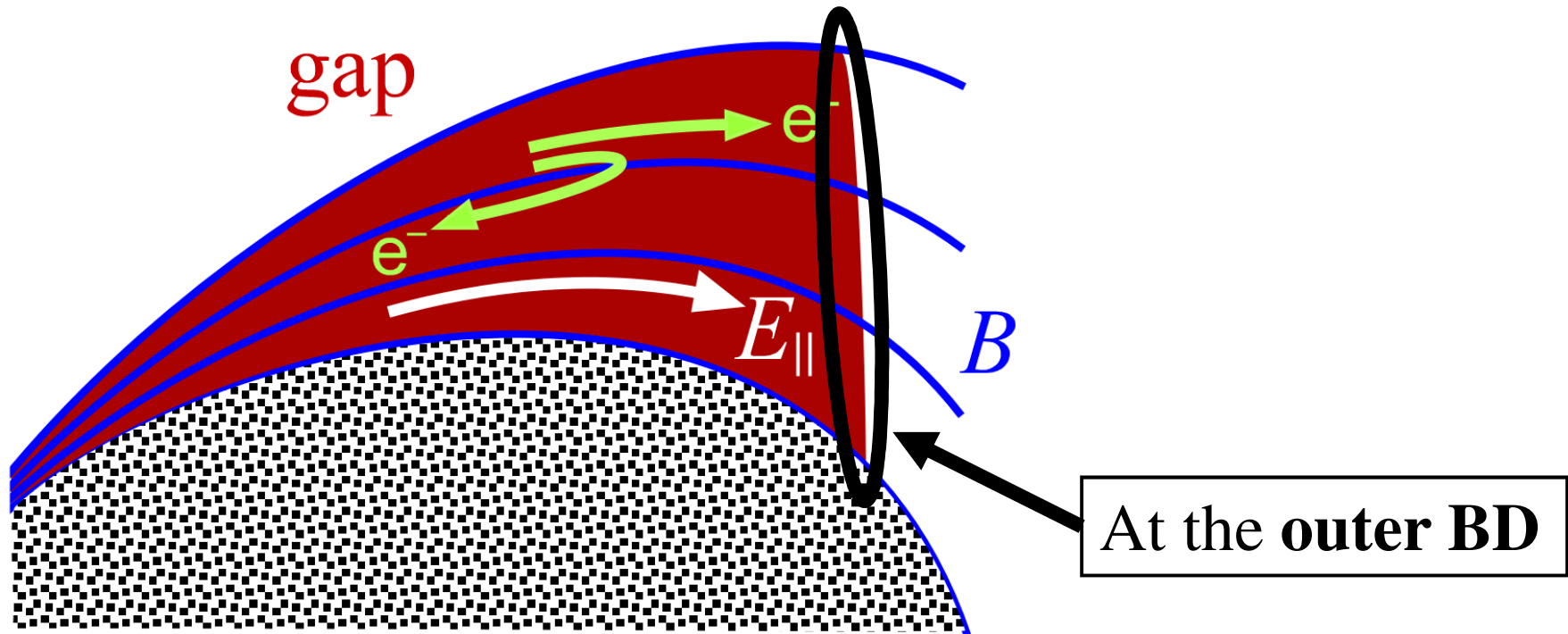
To solve the ODE (radiative transfer eq.), we impose

$$I_\nu(x^{\text{in}}, z, \theta_\gamma) = 0, \quad \text{where } 0 < \theta_\gamma < \pi/2 \text{ (outgoing)}$$

That is, **no e^\pm/γ -ray injection** across the BD.



§3 Modern OG Model: Formalism



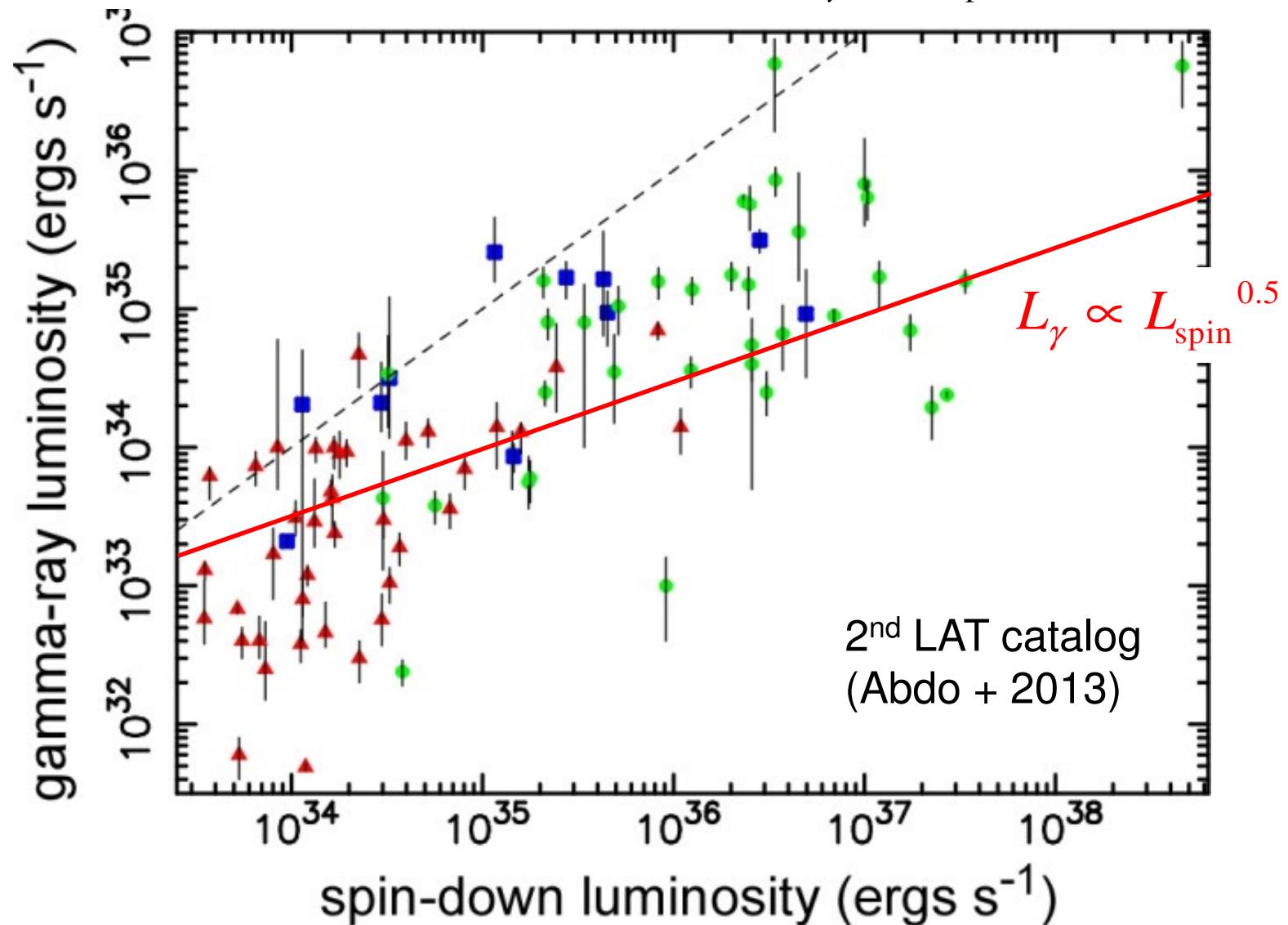
$$N_-(x^{\text{out}}, z, \Gamma) = 0$$

$$I_\nu(x^{\text{out}}, z, \theta_\gamma) = 0, \quad \text{where } \pi/2 < \theta_\gamma < \pi \text{ (in-going)}$$

That is, **no e^\pm/γ -ray injection** across the BD.

§4 Gamma-ray vs. Spin-down Luminosities

First, we demonstrate the observed $L_\gamma \propto L_{\text{spin}}^{0.5}$.



§4 Gamma-ray vs. Spin-down Luminosities

To begin with, let us analytically examine the condition for an OG to be self-sustained. An OG emits the energy flux (KH 2008, ApJ 688, L25)

$$(\nu F_\nu)_{\text{peak}} \approx 0.0450 h_m^3 \frac{\mu^2 \Omega^4}{c^3} \frac{1}{d^2},$$

at distance d by curvature process, where h_m denotes dimensionless OG trans- \mathbf{B} thickness, μ the dipole moment.

OG luminosity can be, therefore, evaluated as

$$L_\gamma \approx 2.36 (\nu F_\nu)_{\text{peak}} \times 4\pi d^2 f_\Omega \approx 1.23 f_\Omega h_m^3 \frac{\mu^2 \Omega^4}{c^3}.$$

Thus, h_m controls the luminosity evolution.

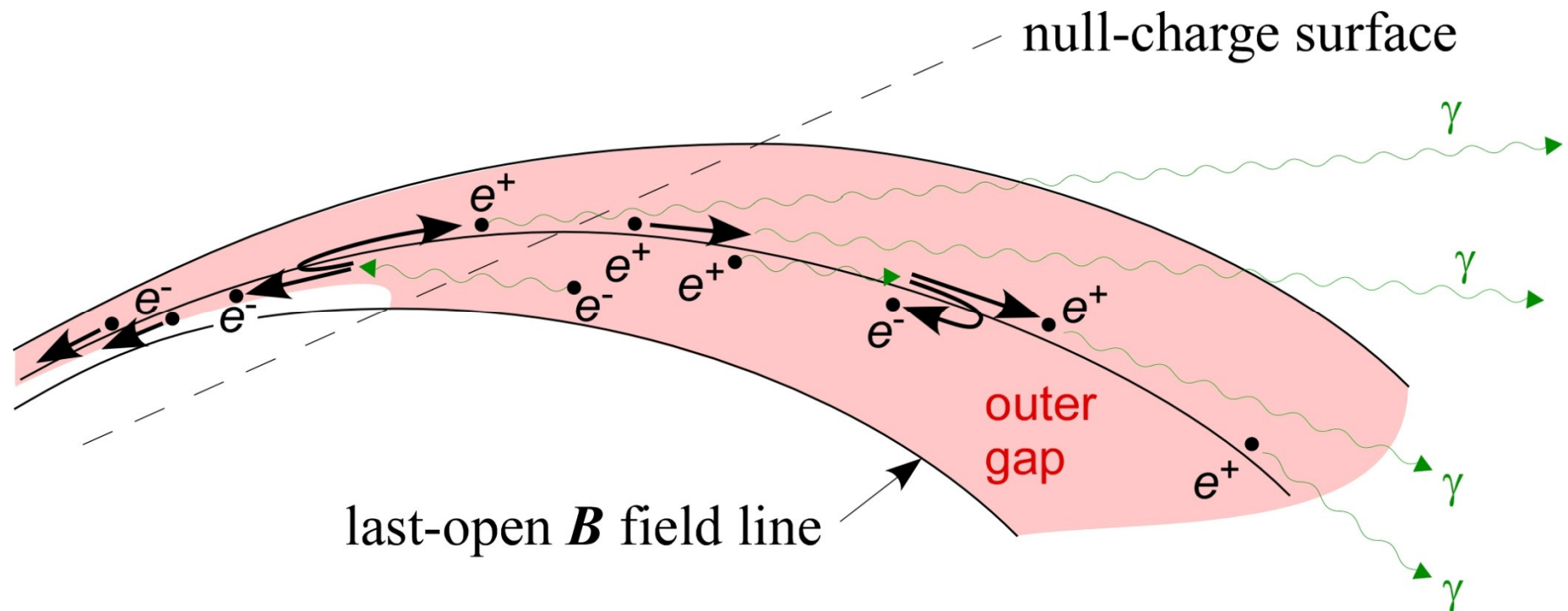
$$\propto \dot{E}$$

§4 Gamma-ray vs. Spin-down Luminosities

To examine h_m , consider the condition of self-sustained OG.

An inward e^- emits $N_\gamma^{\text{in}} \sim 10^4$ synchro-curvature photons,
 $N_\gamma^{\text{in}} \tau^{\text{in}} \sim 10$ of which materialize as pairs.

Each returned, outward e^+ emits $N_\gamma^{\text{out}} \sim 10^5$ curvature photons,
 $N_\gamma^{\text{out}} \tau^{\text{out}} \sim 0.1$ of which materialize as pairs.



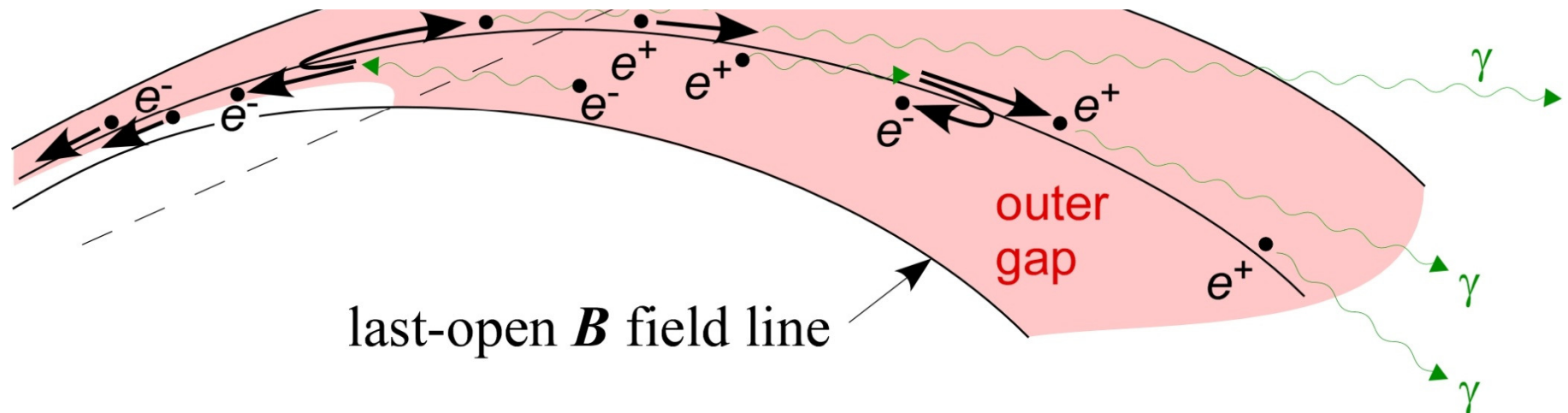
§4 Gamma-ray vs. Spin-down Luminosities

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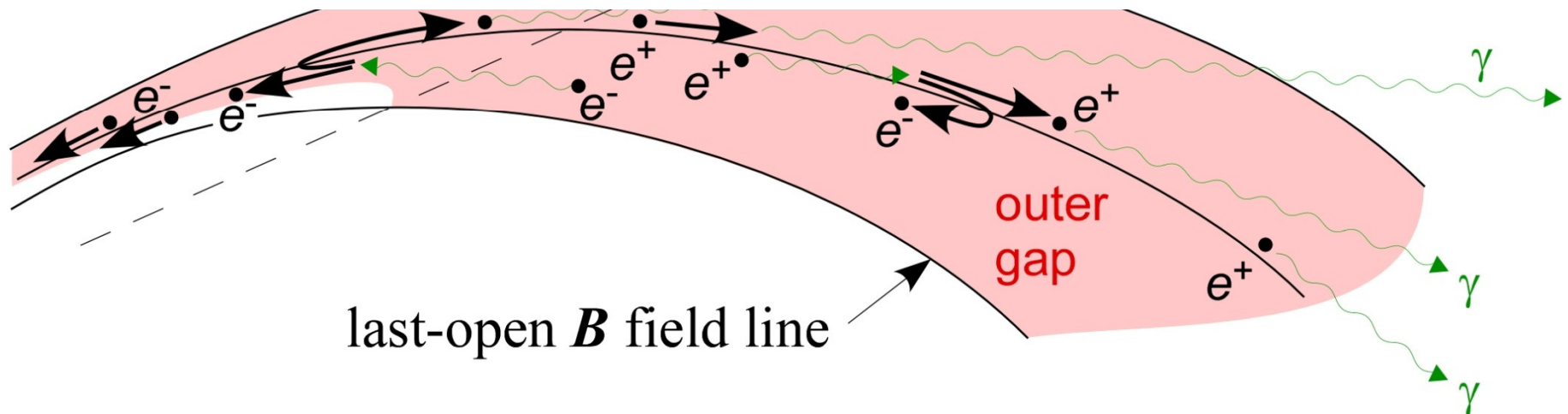
That is, gap trans- \mathbf{B} -field thickness h_m is automatically regulated so that $N_\gamma^{\text{in}} \tau^{\text{in}} N_\gamma^{\text{out}} \tau^{\text{out}} = 1$ is satisfied.



§4 Gamma-ray vs. Spin-down Luminosities

Step 1: Both $N_\gamma^{\text{in}} \tau^{\text{in}}$ and $N_\gamma^{\text{out}} \tau^{\text{out}}$ are expressed in terms of P, μ, α, T , and h_m . Thus, $N_\gamma^{\text{in}} \tau^{\text{in}} N_\gamma^{\text{out}} \tau^{\text{out}} = 1$ gives $h_m = h_m(P, \mu, \alpha, T)$.

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§4 Gamma-ray vs. Spin-down Luminosities

Step 1: express $N_\gamma^{\text{in}} \tau^{\text{in}}$ and $N_\gamma^{\text{out}} \tau^{\text{out}}$ with P, μ, α, T, h_m .

OG model predicts

$$E_{\parallel} \approx \frac{\mu}{2\omega_{\text{LC}}^3} h_m^2.$$

Particles (e^\pm 's) saturate at Lorentz factor,

$$\gamma = \left(\frac{3\rho_c^2}{2e} E_{\parallel} \right)^{1/4},$$

emitting curvature photons with characteristic energy,

$$h\nu_c = \frac{3}{2} \hbar c \frac{\gamma^3}{\rho_c}.$$

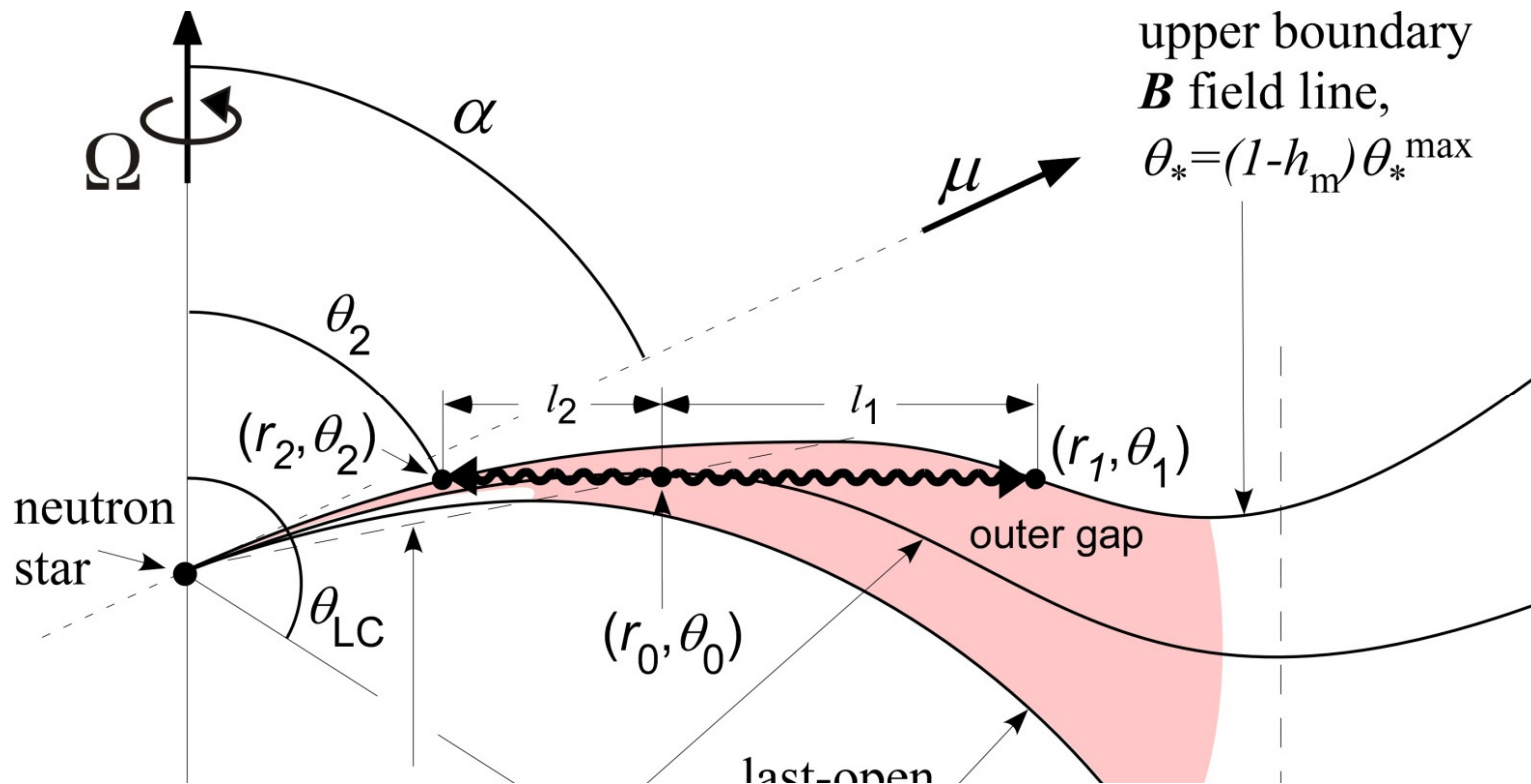
§4 Gamma-ray vs. Spin-down Luminosities

Step 1: express $N_\gamma^{\text{in}} \tau^{\text{in}}$ and $N_\gamma^{\text{out}} \tau^{\text{out}}$ with P, μ, α, T, h_m .

An inward e^- or an outward e^+ emits

$$(N_\gamma)^{\text{in}} = eE_{\parallel} l_2 / h\nu_c, \quad (N_\gamma)^{\text{out}} = eE_{\parallel} l_1 / h\nu_c$$

photons while running the distance l_2 or l_1 .



§4 Gamma-ray vs. Spin-down Luminosities

Step 1: express $N_\gamma^{\text{in}} \tau^{\text{in}}$ and $N_\gamma^{\text{out}} \tau^{\text{out}}$ with P, μ, α, T, h_m .

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photons while running the distance l_2 or l_1 .

Such photons materialize as pairs with probability

$$\tau^{\text{in}} = l_2 F_2 \sigma_2 / c, \quad \tau^{\text{out}} = l_1 F_1 \sigma_1 / c$$

where F_1, F_2 denotes the X-ray flux and σ_1, σ_2 the pair-production cross section.

Quantities $l_1, l_2, F_1, F_2, \sigma_1, \sigma_2$ can be expressed by P, μ, α, T , and h_m , if we specify the \mathbf{B} field configuration.

§4 Gamma-ray vs. Spin-down Luminosities

Step 1: Both $N_\gamma^{\text{in}} \tau^{\text{in}}$ and $N_\gamma^{\text{out}} \tau^{\text{out}}$ are expressed in terms of P, μ, α, T , and h_m . Thus, $N_\gamma^{\text{in}} \tau^{\text{in}} N_\gamma^{\text{out}} \tau^{\text{out}} = 1$ gives $h_m = h_m(P, \mu, \alpha, T)$.

Step 2: Specifying the **spin-down law**, $P = P(t, \alpha)$, and the **cooling curve**, $T = T(t)$, we can solve $h_m = h_m(t, \alpha)$.

Step 3: Independently, $P = P(t, \alpha)$ gives $\dot{E} = \dot{E}(t, \alpha)$.

Step 4: Therefore, we can relate

$$L_\gamma = L_\gamma(t, \alpha) \propto h_m^3 \dot{E}$$

and

$$\dot{E} = \dot{E}(t, \alpha)$$

with intermediate parameter, pulsar age, t .

§4 Gamma-ray vs. Spin-down Luminosities

Step 2: Give spin-down law and NS cooling curve.

Assume dipole-radiation formula,

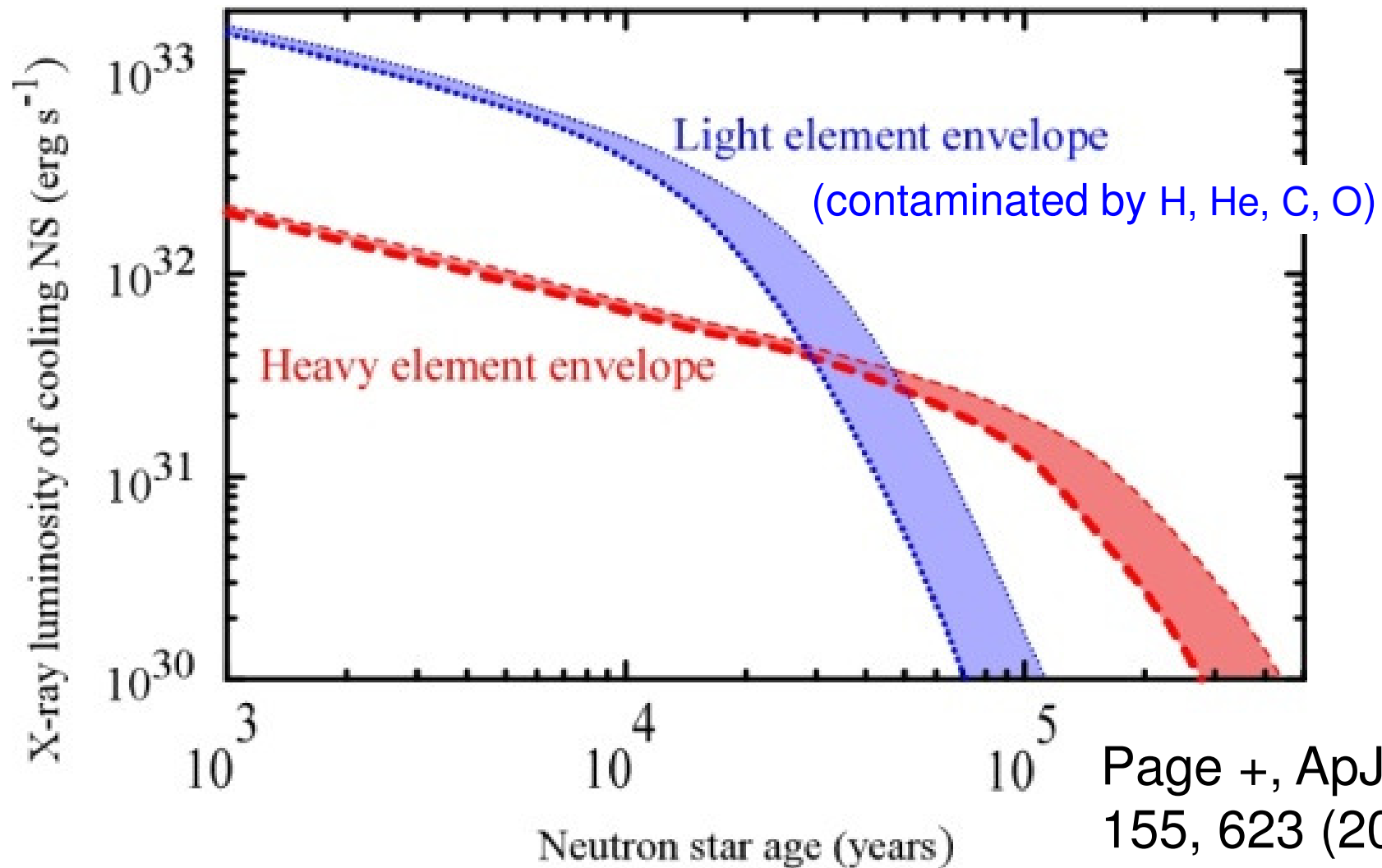
$$-I\Omega\dot{\Omega} = \frac{2}{3} \frac{\mu^2 \Omega^4}{c^3} \quad \rightarrow \quad P = P(t, \alpha)$$

Adopt the minimum cooling scenario (i.e., without any direct-Urca, rapid cooling processes).

$$\rightarrow T = T(t)$$

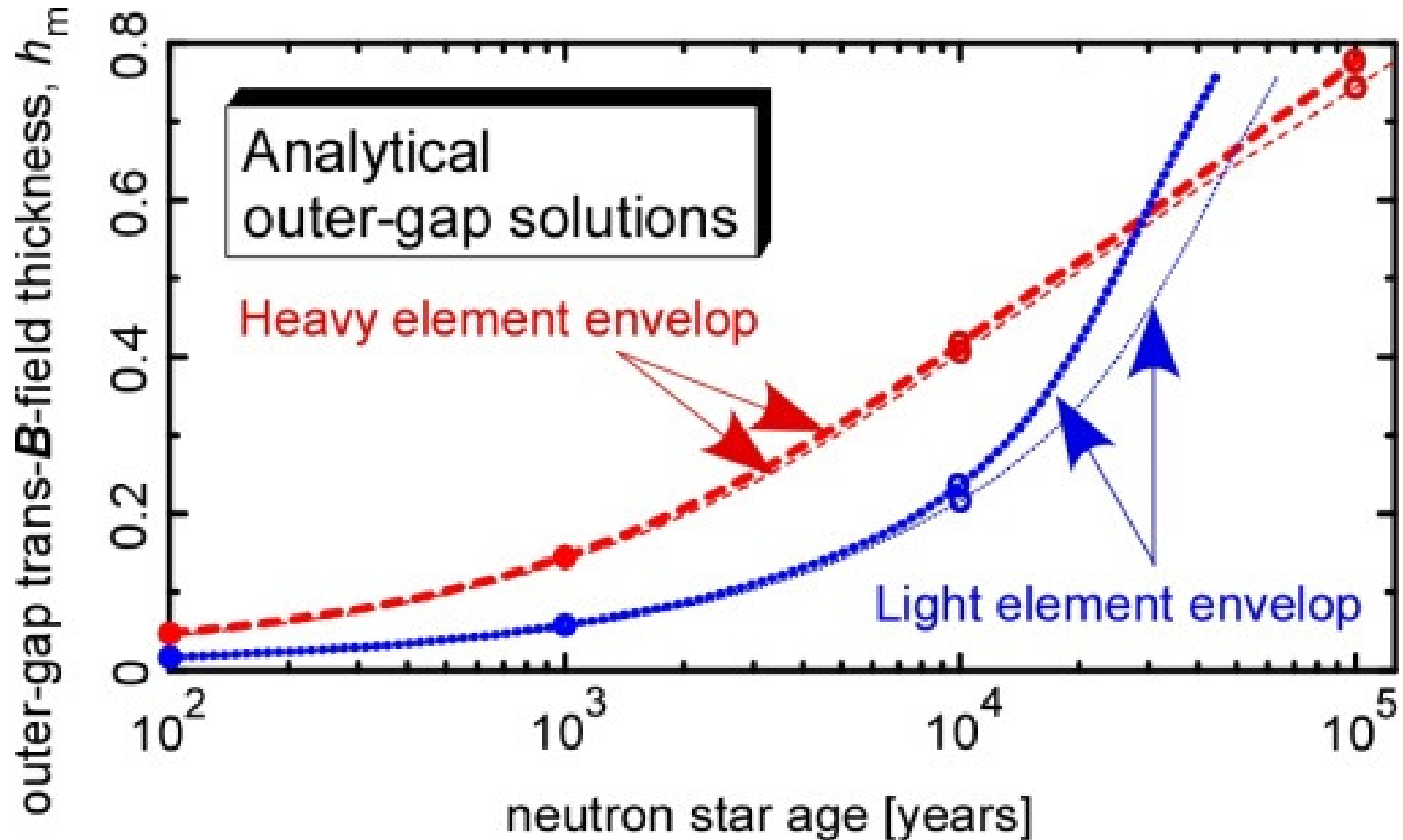
§4 Gamma-ray vs. Spin-down Luminosities

Step 2: Cooling curves in the minimum cooling scenario:



§4 Gamma-ray vs. Spin-down Luminosities

Step 2: Now we can solve $h_m = h_m(t)$.



§4 Gamma-ray vs. Spin-down Luminosities

Step 1: Both $N_\gamma^{\text{in}} \tau^{\text{in}}$ and $N_\gamma^{\text{out}} \tau^{\text{out}}$ are expressed in terms of P, μ, α, T , and h_m . Thus, $N_\gamma^{\text{in}} \tau^{\text{in}} N_\gamma^{\text{out}} \tau^{\text{out}} = 1$ gives $h_m = h_m(P, \mu, \alpha, T)$.

Step 2: Specifying the spin-down law, $P = P(t, \alpha)$, and the cooling curve, $T = T(t)$, we can solve $h_m = h_m(t, \alpha)$.

Step 3: On the other hand, $P = P(t, \alpha)$ gives $\dot{E} = \dot{E}(t, \alpha)$.

Step 4: Therefore, we can relate

$$L_\gamma = L_\gamma(t, \alpha) \propto h_m^3 \dot{E}$$

and

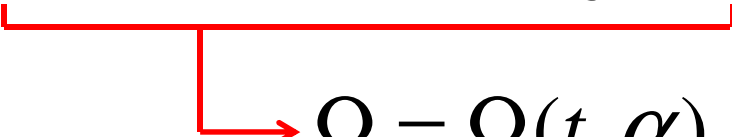
$$\dot{E} = \dot{E}(t, \alpha)$$


with intermediate parameter, pulsar age, t .

§4 Gamma-ray vs. Spin-down Luminosities

Step 3: We can immediately solve $\dot{E} = \dot{E}(t, \alpha)$ by the spin-down law.

$$\dot{E} = -I\Omega\dot{\Omega} = C(\alpha) \frac{\mu^2 \Omega^4}{c^3}$$


$$\Omega = \Omega(t, \alpha)$$


$$\dot{E} = \dot{E}(t, \alpha)$$

$$C = \frac{2}{3} \sin^2 \alpha \quad \text{for magnetic-dipole spin-down}$$

$$\approx 1 + \sin^2 \alpha \quad \text{for force-free spin-down}$$

§4 Gamma-ray vs. Spin-down Luminosities

Step 1: Both $N_\gamma^{\text{in}} \tau^{\text{in}}$ and $N_\gamma^{\text{out}} \tau^{\text{out}}$ are expressed in terms of P, μ, α, T , and h_m . Thus, $N_\gamma^{\text{in}} \tau^{\text{in}} N_\gamma^{\text{out}} \tau^{\text{out}} = 1$ gives $h_m = h_m(P, \mu, \alpha, T)$.

Step 2: Specifying the spin-down law, $P = P(t, \alpha)$, and the cooling curve, $T = T(t)$, we can solve $h_m = h_m(t, \alpha)$.

Step 3: On the other hand, $P = P(t, \alpha)$ gives $\dot{E} = \dot{E}(t, \alpha)$.

Step 4: Therefore, we can relate

$$L_\gamma = L_\gamma(t, \alpha) \propto h_m^3 \dot{E}$$

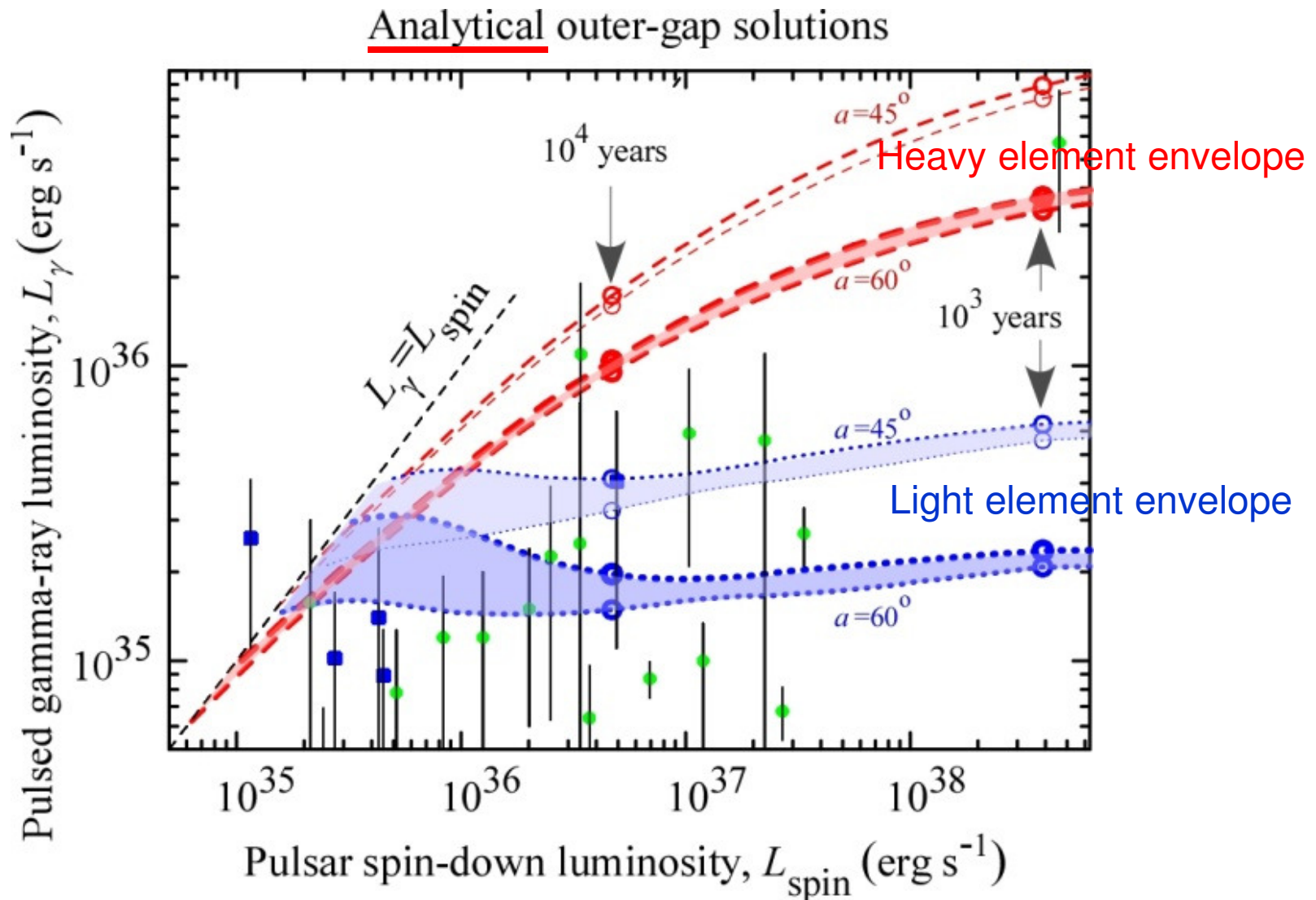
and

$$\dot{E} = \dot{E}(t, \alpha)$$

with intermediate parameter, pulsar age, t .

§4 Gamma-ray vs. Spin-down Luminosities

Step 4: Use $h_m = h_m(t)$ to relate $L_\gamma \propto h_m^3 \dot{E}$ with $\dot{E} = \dot{E}(t)$.

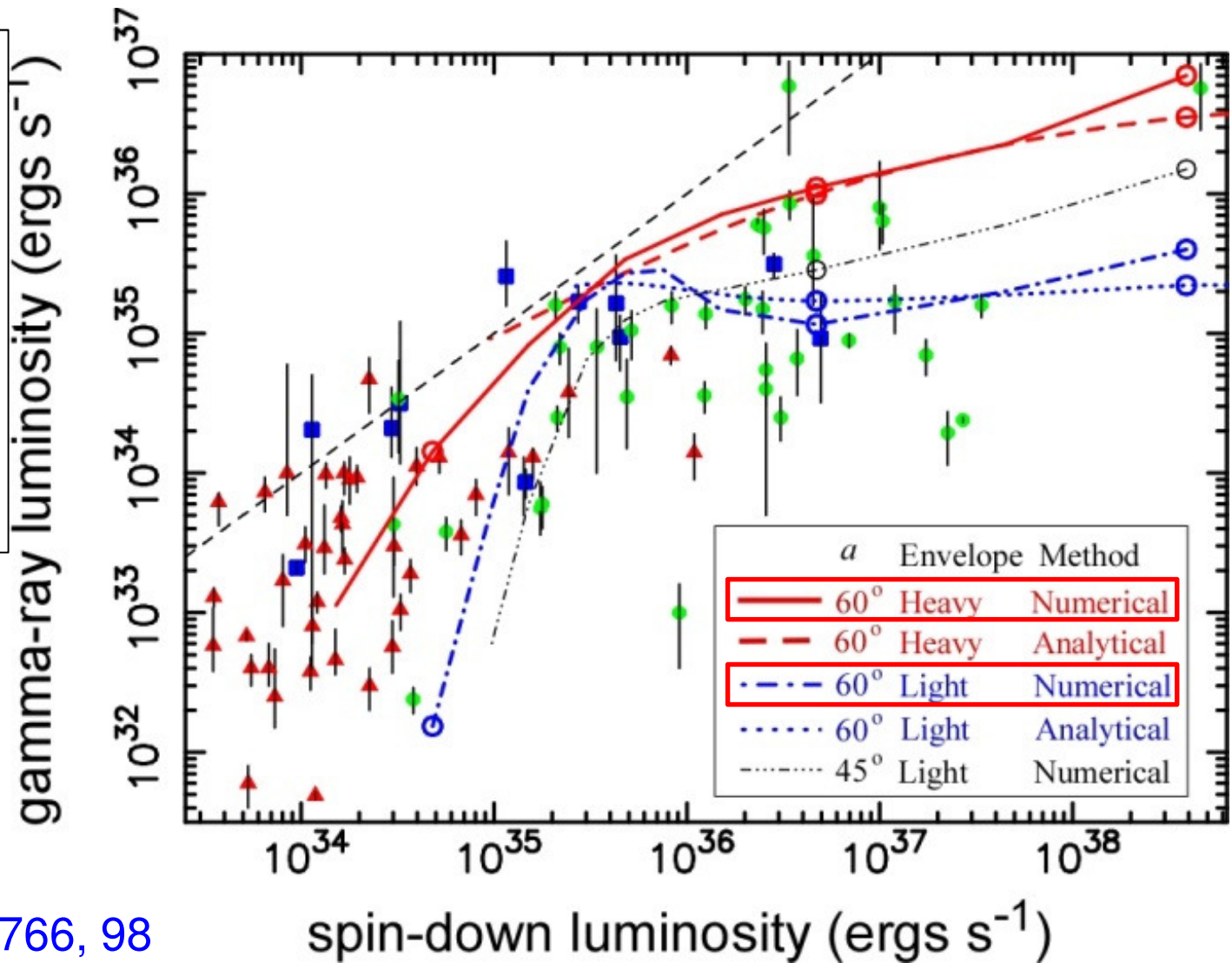


§4 Gamma-ray vs. Spin-down Luminosities

Numerical solution is consistent with the analytical one.

$L_\gamma \sim L_{\text{spin}}^{0-0.4}$
if $L_{\text{spin}} > 10^{36}$
erg/s

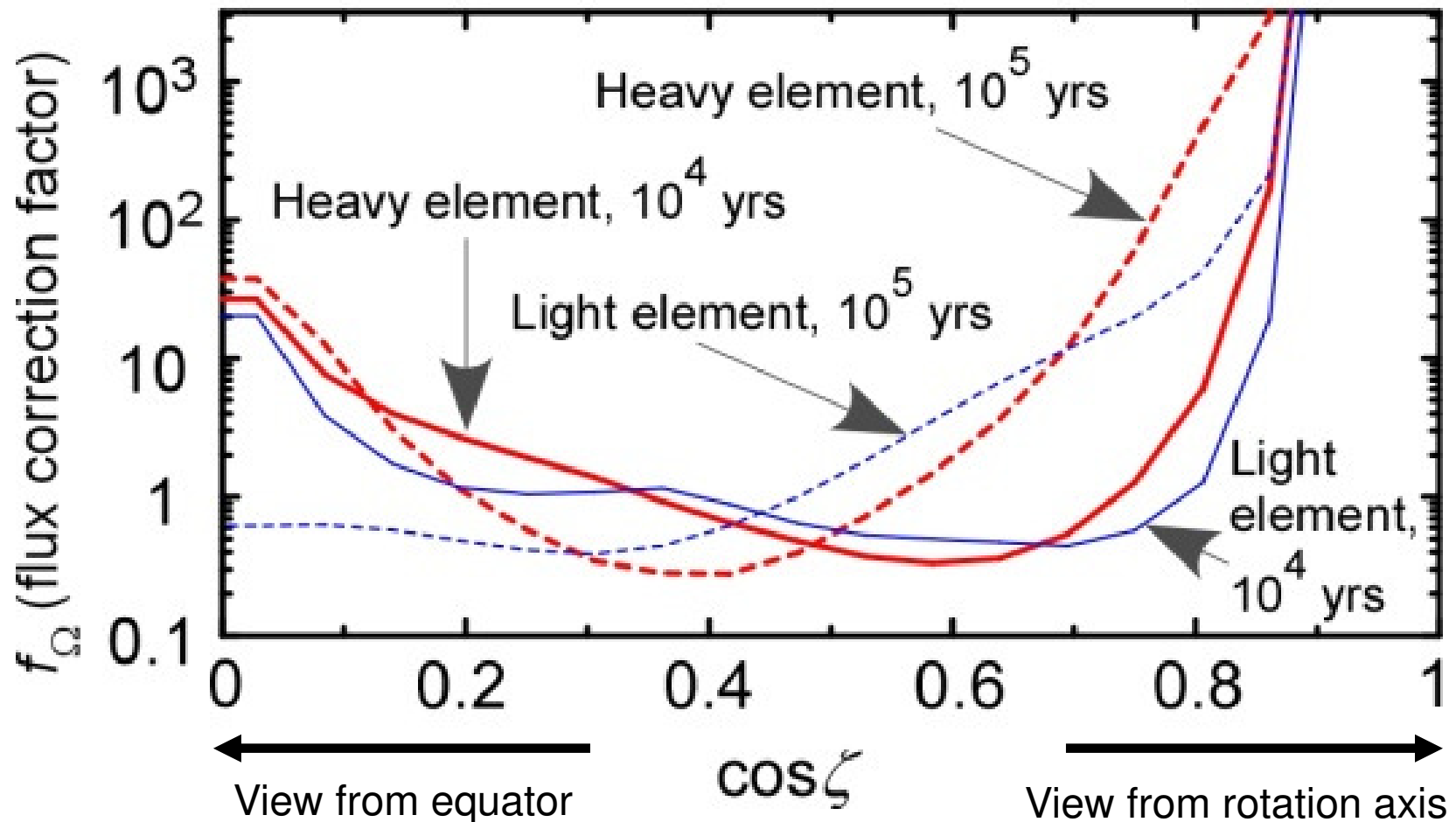
However,
 L_γ declines
rapidly
if $L_{\text{spin}} < 10^{35.5}$
erg/s



§4 Gamma-ray vs. Spin-down Luminosities

To convert the observed flux F_γ into luminosity, $L_\gamma = 4\pi f_\Omega F_\gamma d^2$, it is (conventionally) assumed $f_\Omega = 1$.

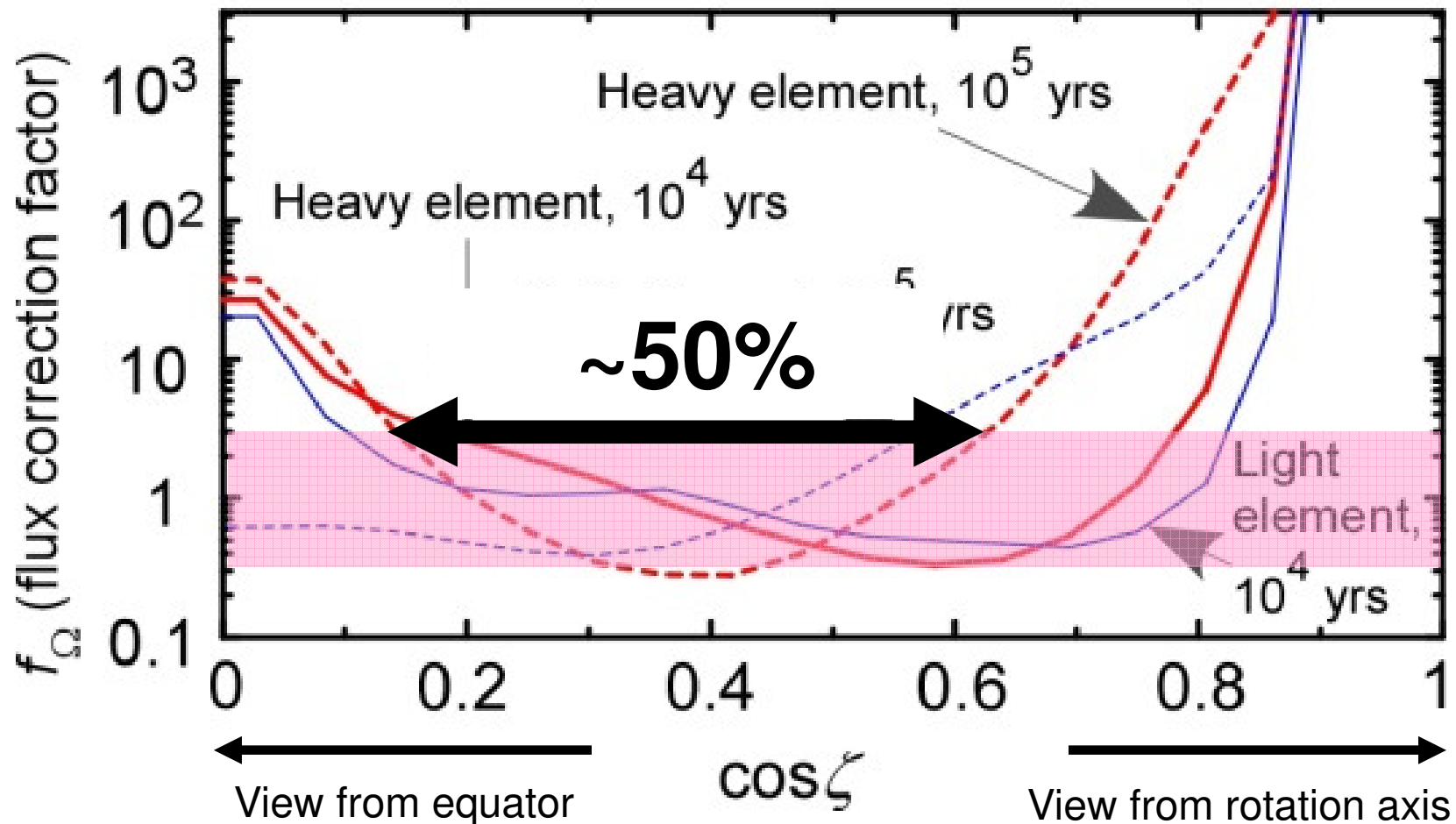
However, we **underestimate L_γ if $f_\Omega > 1$** .



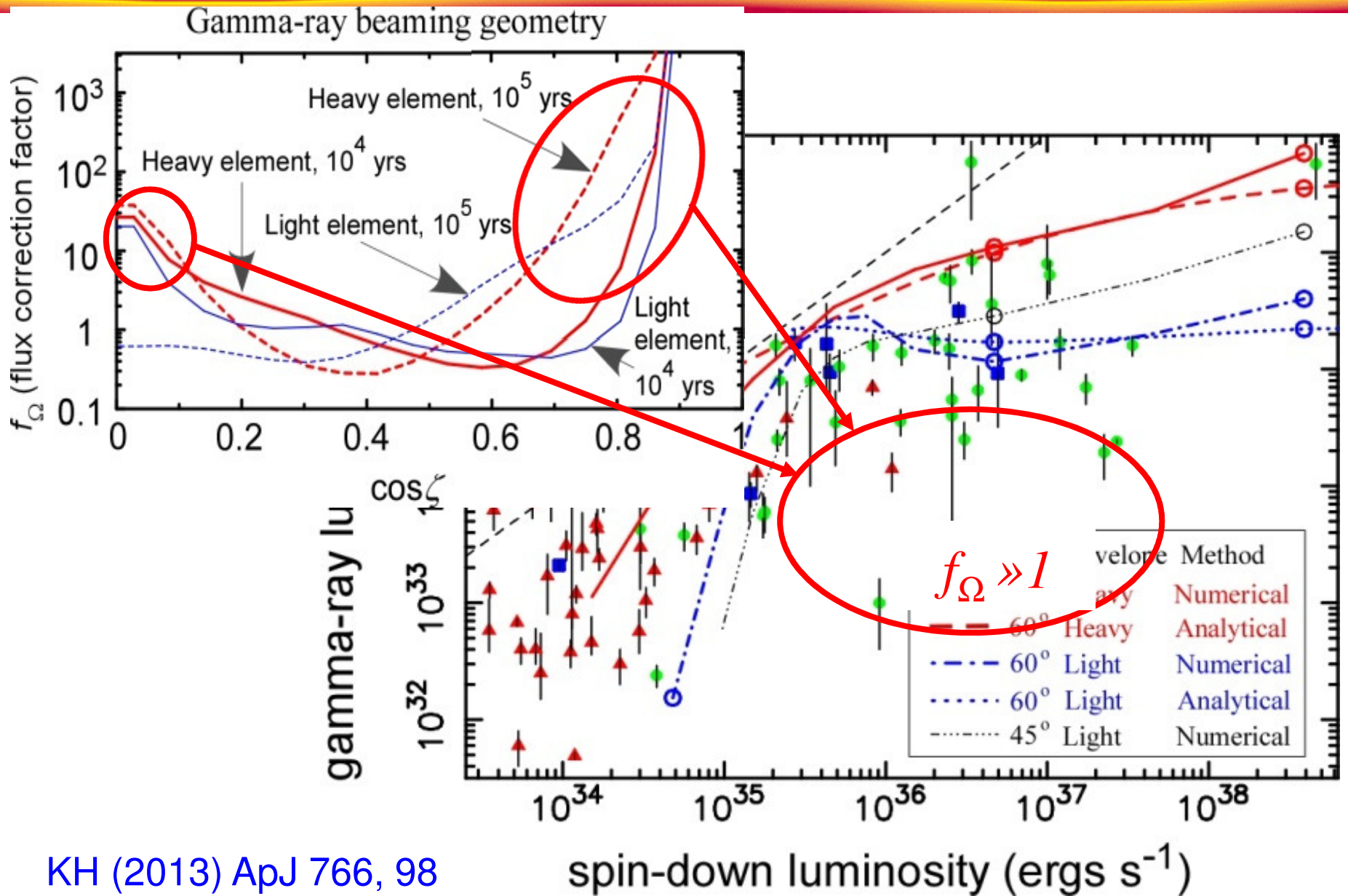
§4 Gamma-ray vs. Spin-down Luminosities

To convert the observed flux F_γ into luminosity, $L_\gamma = 4\pi f_\Omega F_\gamma d^2$, it is (conventionally) assumed $f_\Omega = 1$.

For example, we obtain $f_\Omega > 3$ with probability $\sim 50\%$.



§4 Gamma-ray vs. Spin-down Luminosities



KH (2013) ApJ 766, 98

§5 Application to the Crab pulsar

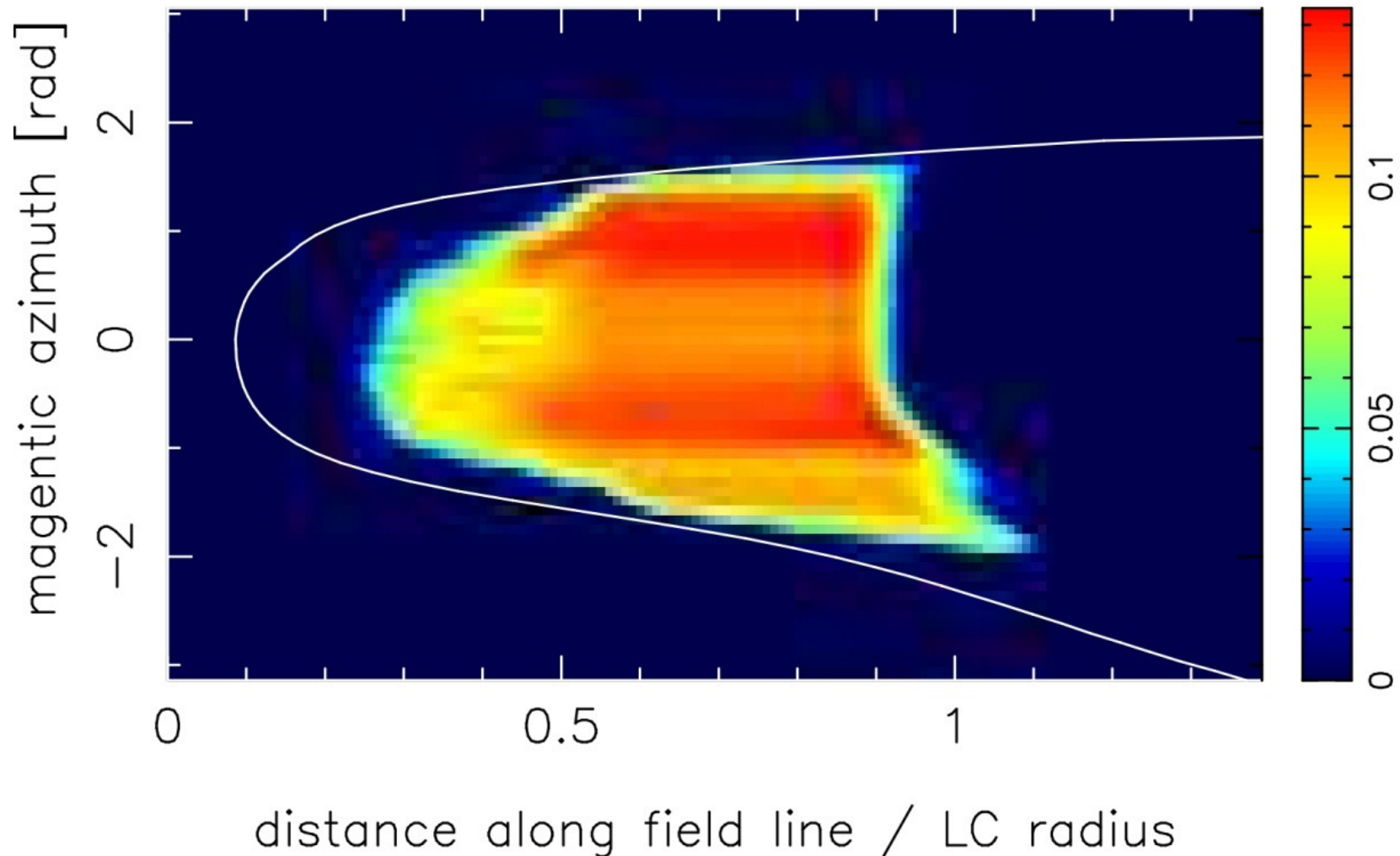
We can apply the same numerical scheme to the Crab pulsar.

Today, we assume

- magnetic inclination angle $\alpha=60^\circ$,
- cooling NS surface temperature $kT=100\text{eV}$,
(consistent with the cooling curve of a heavy-element envelope)

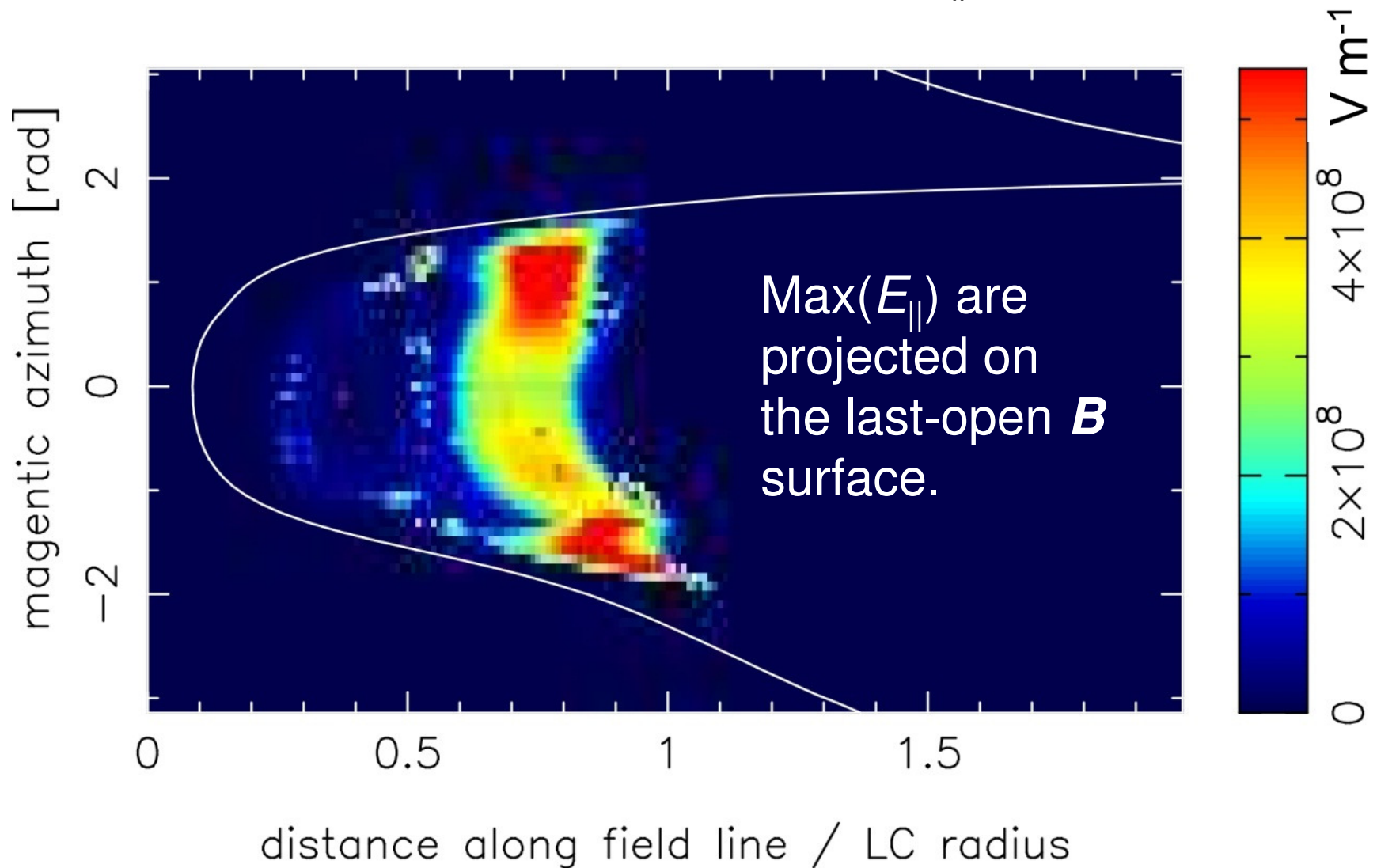
§5 Application to the Crab pulsar

3-D OG distribution: trans- \mathbf{B} thickness, D_{\perp} ,
projected on the last-open \mathbf{B} field line surface.

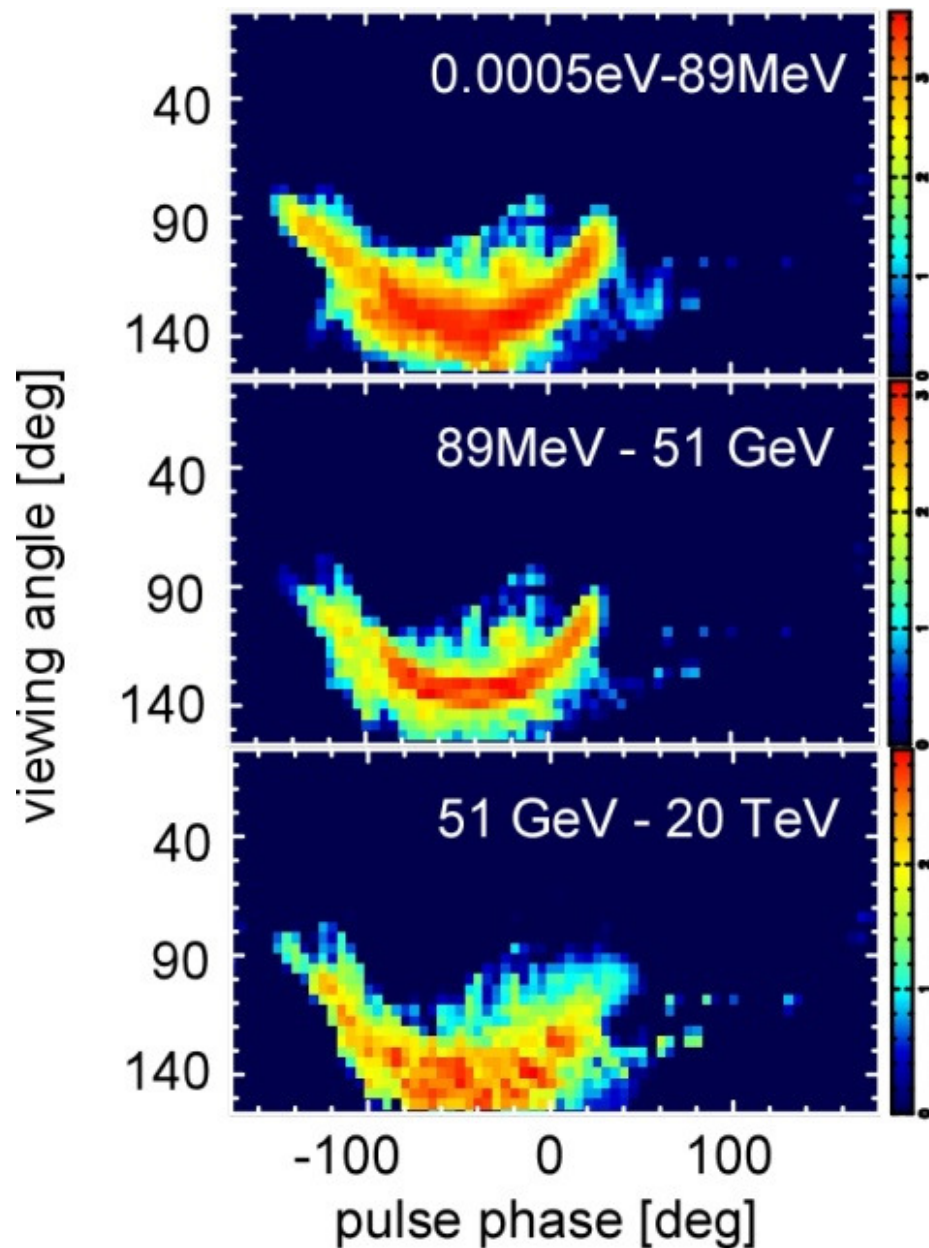


§5 Application to the Crab pulsar

Distribution of acceleration E field, E_{\parallel} .



§5 Application to the Crab pulsar



Sky map of OG emission

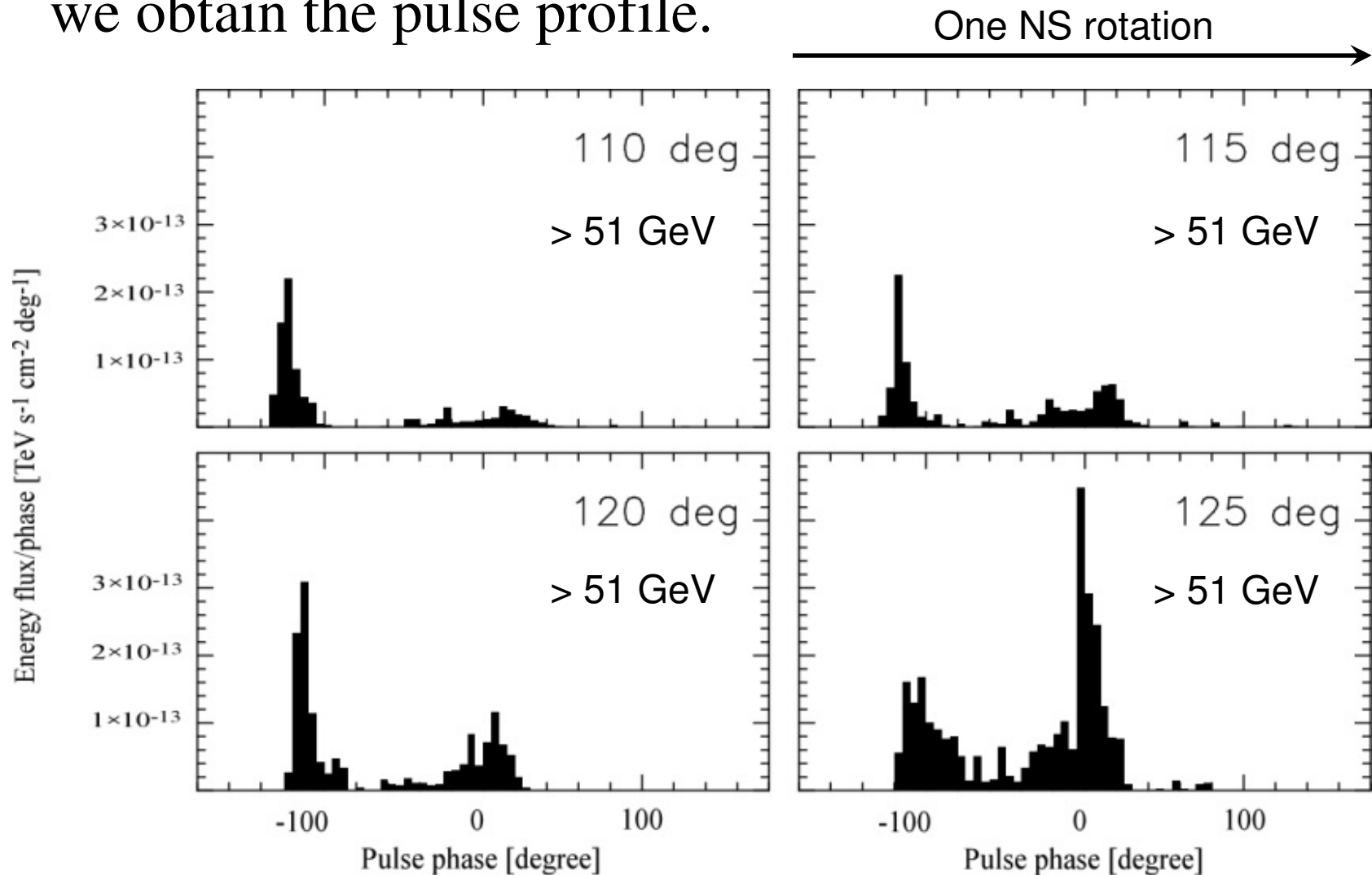
Secondary/tertiary
synchrotron emission

Primary **curvature** &
secondary/tertiary
SSC emission

Secondary/tertiary
SSC emission

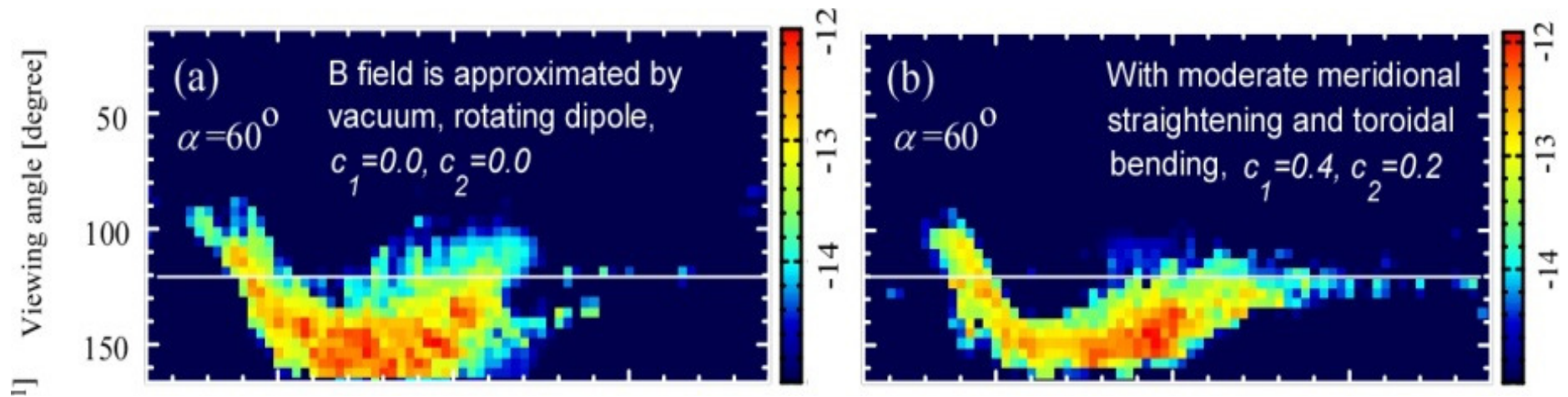
§5 Application to the Crab pulsar

If we cut the sky map at a specific viewing angle, we obtain the pulse profile.



§5 Application to the Crab pulsar

From X-ray observations (of the Crab nebula),
 $\zeta \sim 120^\circ$ is suggested.



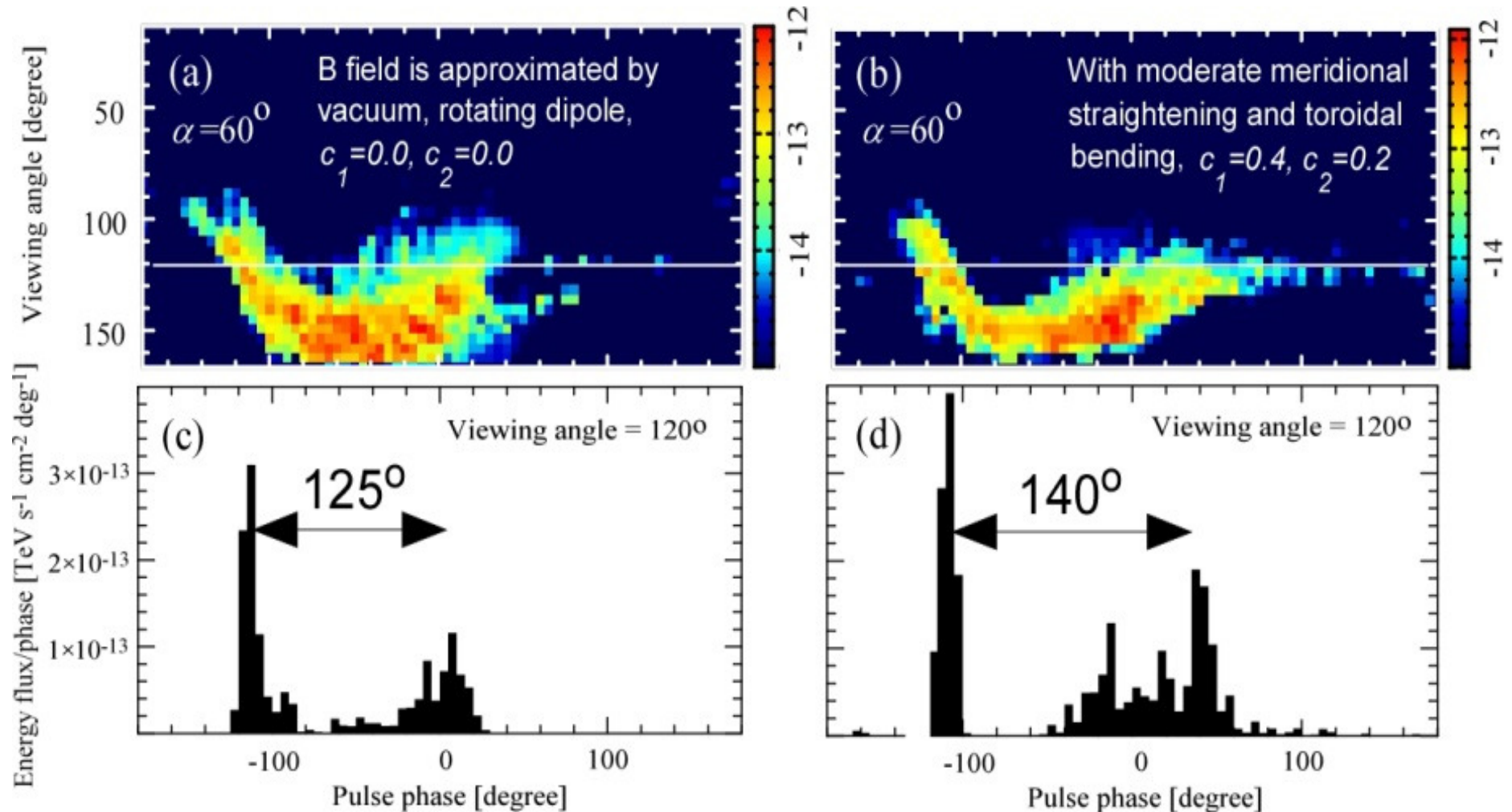
Introduce artificial meridional straightening and toroidal bending of \mathbf{B} field (due to current):

$$B_\theta = (1 - c_1 \varpi / \varpi_{\text{LC}}) B_{\theta, \text{vac}}$$

$$B_\phi = (1 + c_2 \varpi / \varpi_{\text{LC}}) B_{\phi, \text{vac}}$$

§5 Application to the Crab pulsar

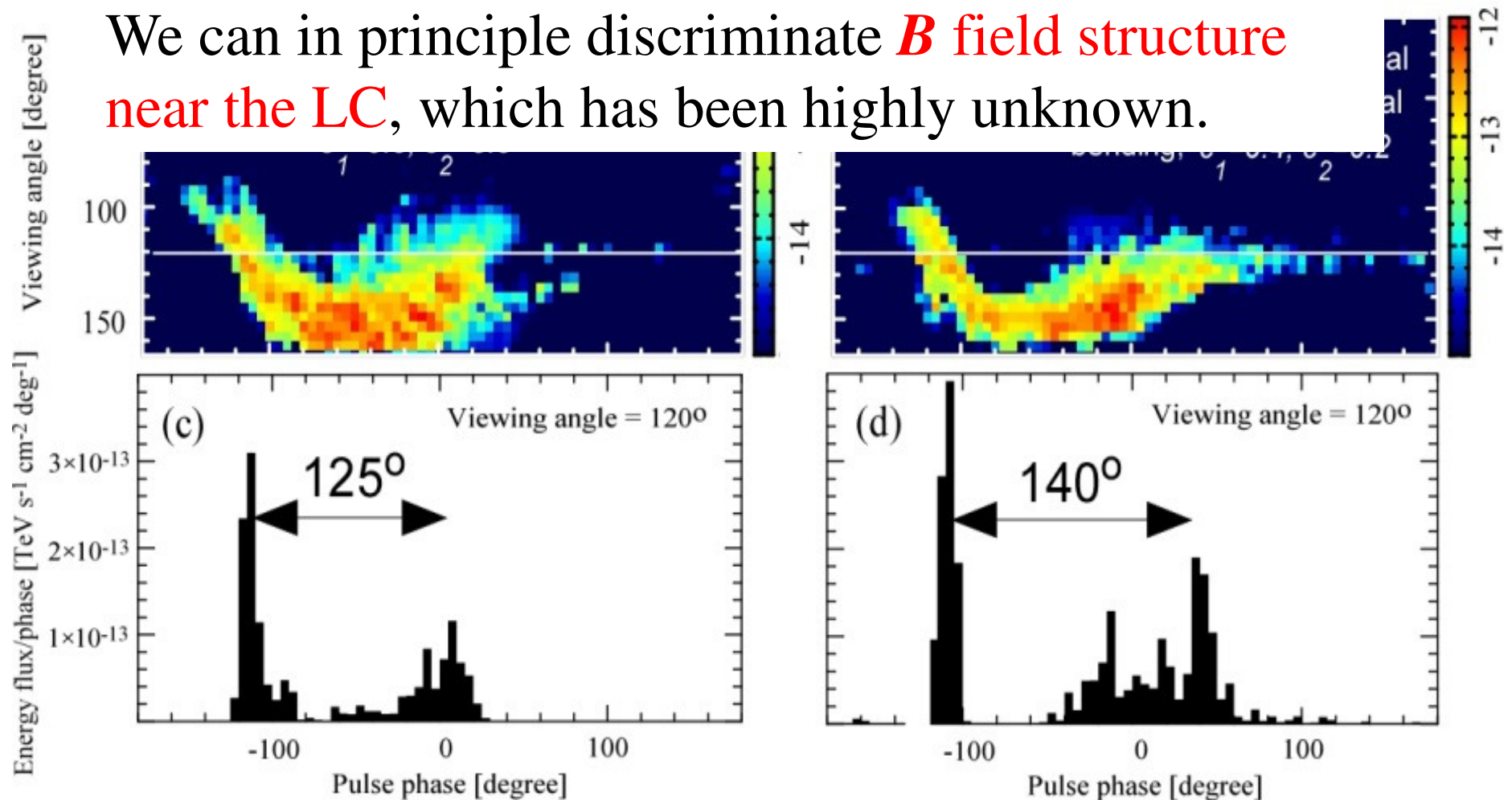
Peak separation increases if B field is toroidally bent and meridionally straightened moderately.



§5 Application to the Crab pulsar

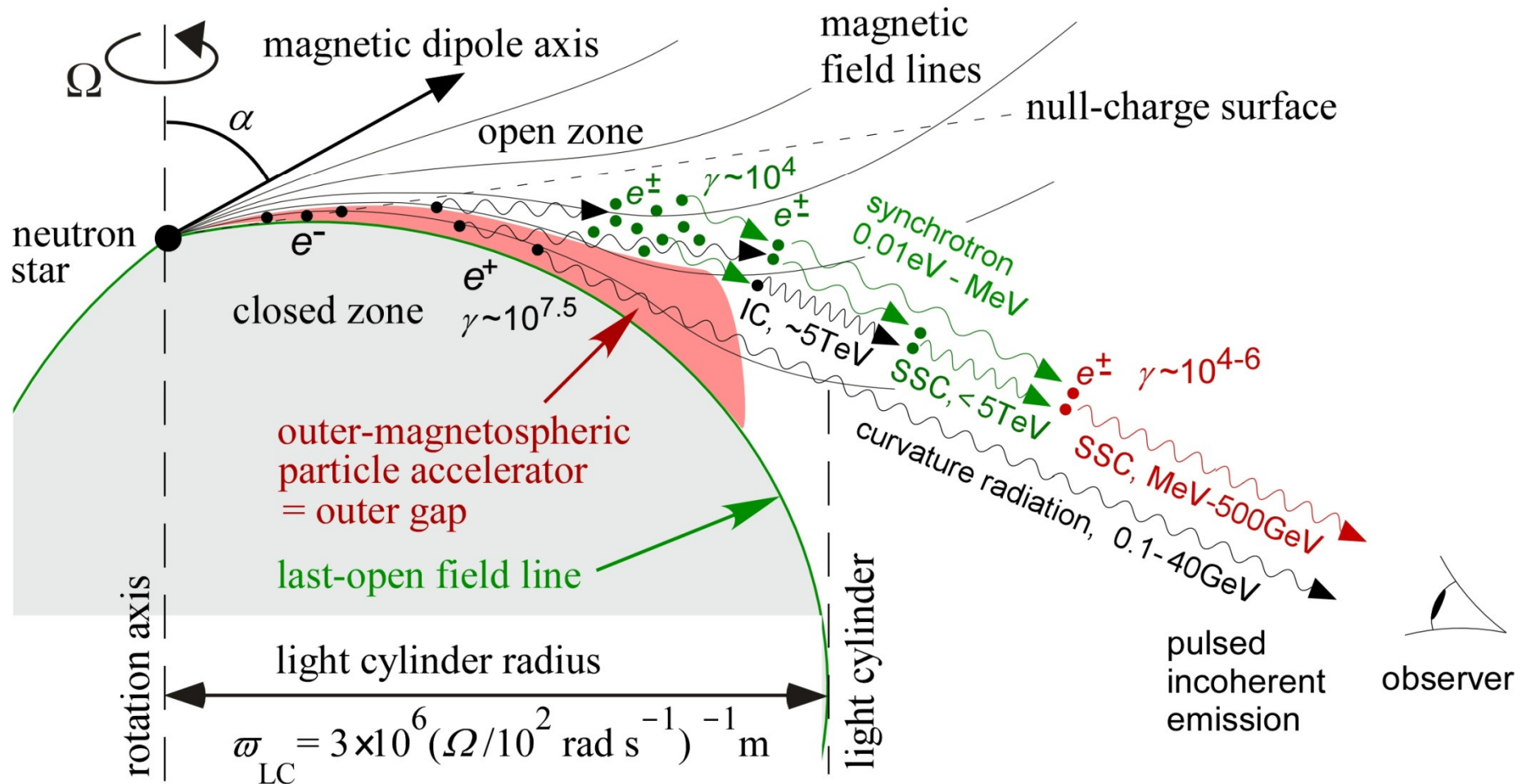
Peak separation increases if B field is toroidally bent and meridionally straightened moderately.

We can in principle discriminate B field structure near the LC, which has been highly unknown.



§5 Application to the Crab pulsar

Schematic picture of cascading pairs and their emissions:



§6 Lorentz invariance violation tests

Finally, let us consider the Lorentz invariance violation tests using pulsars.

Quantum gravity can be tested e.g., by measuring an energy-dependent dispersion relation of mass-less particles.

Ex.) Photons would propagate at the speed

$$\frac{v(E)}{c} = 1 \pm \left(\frac{E}{E_{QG}} \right)^n$$

For $n=1$, for instance, two photons with different energies E_1 and E_2 will arrive with time difference,

$$\Delta t = \frac{L}{c} \frac{E_2 - E_1}{E_{QG}}$$

§6 Lorentz invariance violation tests

$$\Delta t = \frac{L}{c} \frac{E_2 - E_1}{E_{\text{QG}}} \quad \text{or} \quad E_{\text{QG}} = \frac{L}{c} \frac{E_2 - E_1}{\Delta t}$$

If two photons are emitted at the same place (i.e., same L), we can derive E_{QG} (or set a lower bound of E_{QG}) from Δt .

Shot-time events (small Δt) with large photon-energy separation (greater $E_2 - E_1$) are ideal.

§6 Lorentz invariance violation tests

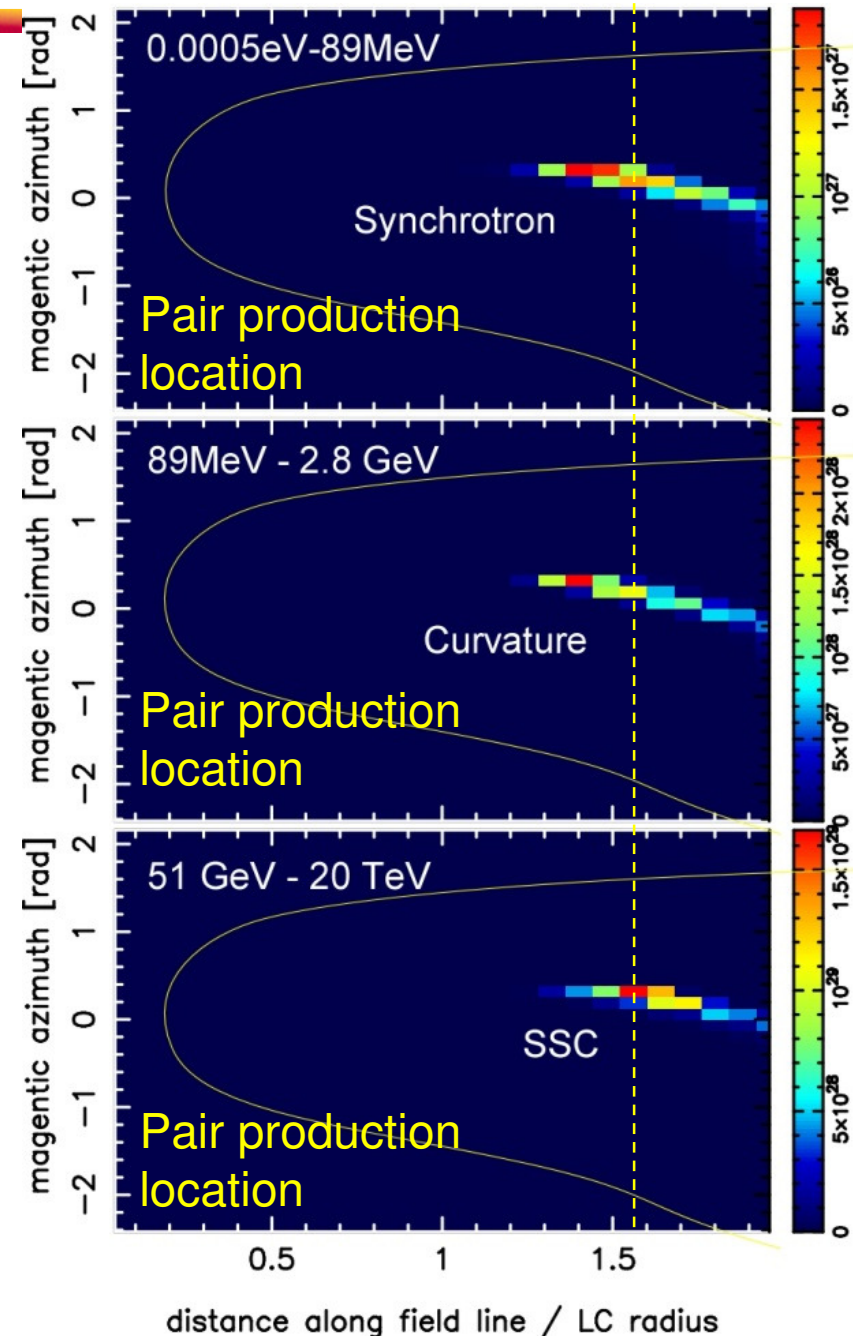
They applied this method to rotation-powered pulsars and examined Δt at different photon energies, assuming that **GeV** and **TeV** photons are emitted from the **same location** in the magnetosphere.

See e.g.,

Zitser +, ICRC2013, Rio de Janeiro

Otte, ICRC, Beijing

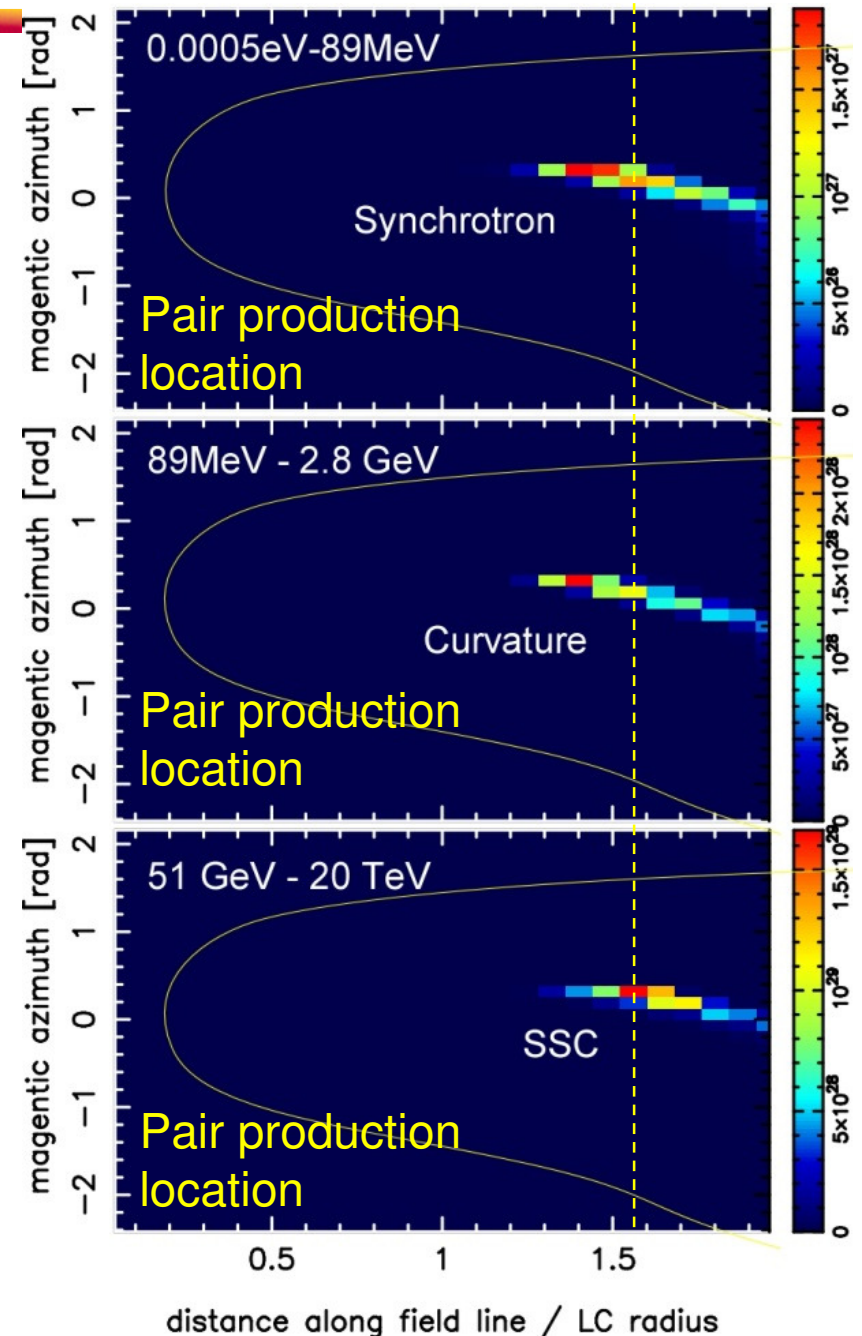
McCann +, 4th Fermi sympo, Monteley



§6 Lorentz invariance violation tests

They applied this method to rotation-powered pulsars and examined Δt at different photon energies, assuming that **GeV** and **TeV** photons are emitted from the **same location** in the magnetosphere.

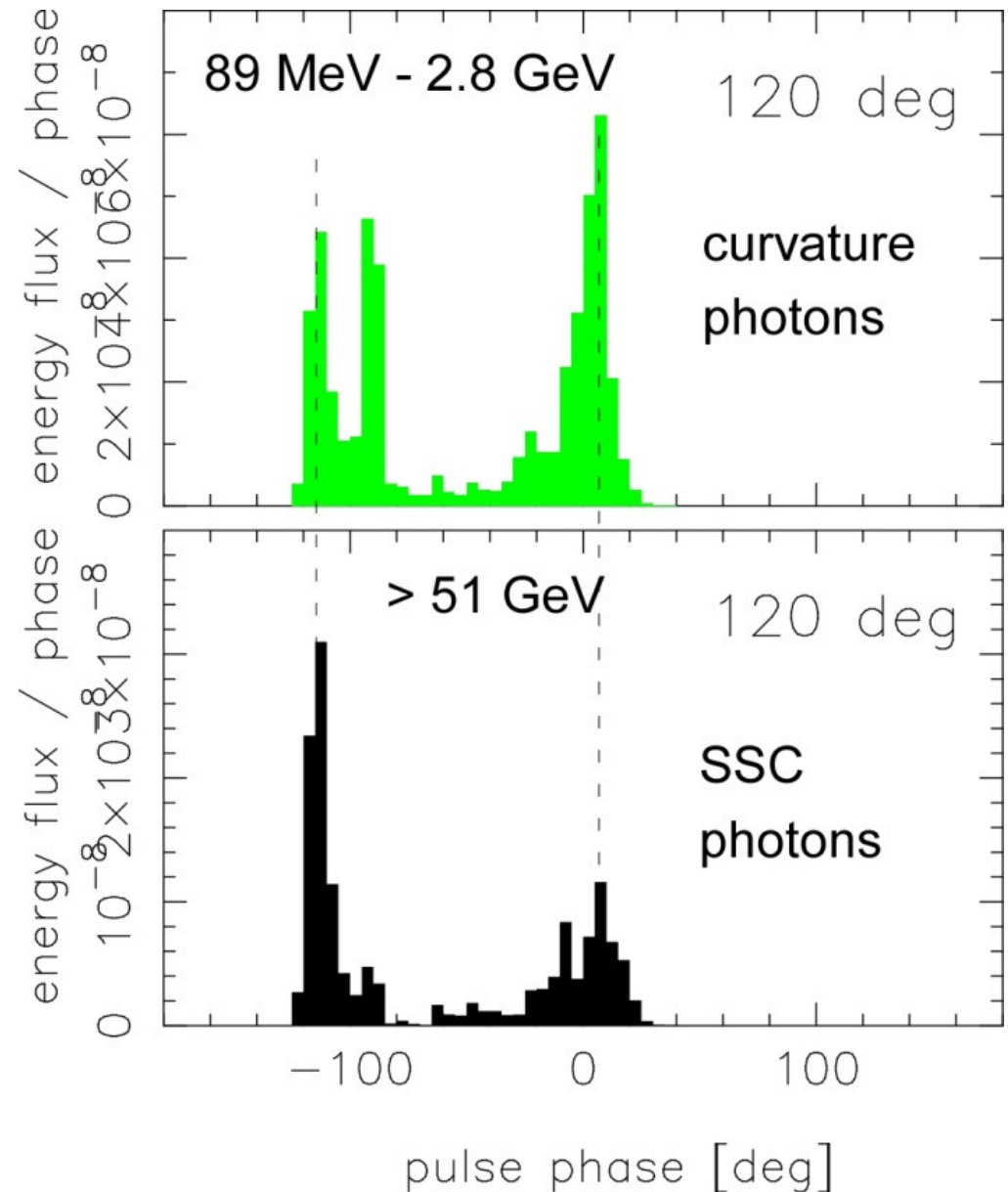
However, **higher energy photons** are emitted from **higher altitudes** by **different emission mechanisms**.



§6 Lorentz invariance violation tests

Indeed, **curvature** photons and **SSC** ones appear close in phase.

However, if they appear in different phases, it may merely mean that they are **emitted from different locations** in the magnetosphere.



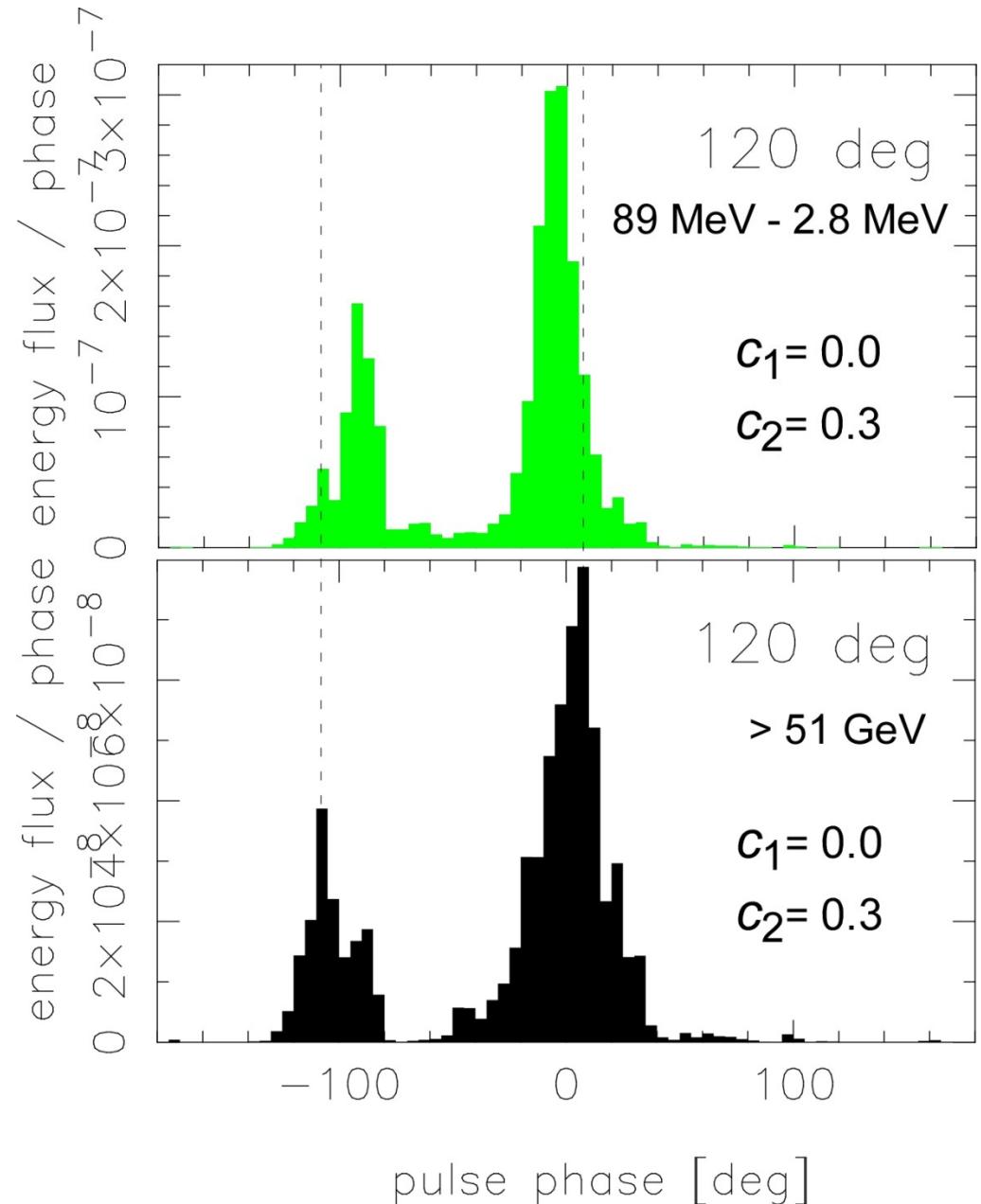
§6 Lorentz invariance violation tests

For example, in B field is moderately bent toroidally in the counter-rotation direction, **HE and VHE pulses** will arrive at **different phase**.

This has nothing to do with the LIV.

$$B_{\theta} = (1 - c_1 \bar{\omega} / \bar{\omega}_{LC}) B_{\theta, \text{vac}}$$

$$B_{\phi} = (1 + c_2 \bar{\omega} / \bar{\omega}_{LC}) B_{\phi, \text{vac}}$$



Summary

- High-energy pulsar observations have made rapid progress in recent five years by the advent of Fermi/LAT.
- Development of pulsar emission theory is highly required.
- Now we can predict the HE emissions from pulsar outer magnetospheres, by solving the set of Maxwell ($\text{div}\mathbf{E}=4\pi\rho$) and Boltzmann eqs., if we specify P , dP/dt , α_{incl} , kT_{NS} .
- The solution coincidentally corresponds to a quantitative extension of classical outer gap model. However, **we no longer have to assume the gap geometry, E_{\parallel} , e^{\pm} distribution functions.**
- γ -ray luminosity evolves as $L_{\gamma} \propto \dot{E}^{0-0.4}$ when $\dot{E} > 10^{36.5} \text{ erg s}^{-1}$, which is consistent with Fermi/LAT observations.
- Crab pulsar's phase-resolved spectrum can be explained by the current outer-magnetospheric accelerator theory.



Thank you.